Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 913 (2016) 792–814

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

The $\Delta(27)$ flavor 3-3-1 model with neutral leptons

V.V. Vien [a,b], A.E. Cárcamo Hernández [c,*], H.N. Long [d]

[a] *Institute of Research and Development, Duy Tan University, 182 Nguyen Van Linh, Da Nang City, Viet Nam*
[b] *Department of Physics, Tay Nguyen University, 567 Le Duan, Buon Ma Thuot, DakLak, Viet Nam*
[c] *Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile*
[d] *Institute of Physics, Vietnam Academy of Science and Technology, 10 Dao Tan, Ba Dinh, Hanoi, Viet Nam*

Received 15 September 2016; received in revised form 12 October 2016; accepted 13 October 2016

Available online 19 October 2016

Editor: Tommy Ohlsson

Abstract

We build the first 3-3-1 model based on the $\Delta(27)$ discrete group symmetry, consistent with fermion masses and mixings. In the model under consideration, the neutrino masses are generated from a combination of type-I and type-II seesaw mechanisms mediated by three heavy right-handed Majorana neutrinos and three $SU(3)_L$ scalar antisextets, respectively. Furthermore, from the consistency of the leptonic mixing angles with their experimental values, we obtain a non-vanishing leptonic Dirac CP violating phase of $-\frac{\pi}{2}$. Our model features an effective Majorana neutrino mass parameter of neutrinoless double beta decay, with values $m_{\beta\beta} = 10$ and 18 meV for the normal and the inverted neutrino mass hierarchies, respectively.

1. Introduction

The discovery of the 126 GeV Higgs boson at the Large Hadron Collider (LHC) [1,2], has filled the vacancy of the Higgs boson needed for the completion of the Standard Model (SM) at the Fermi scale and has provided a confirmation for the mass generation mechanism of the weak gauge bosons. Despite LHC experiments indicating that the decay modes of the new scalar state

* Corresponding author.
E-mail addresses: wvienk16@gmail.com (V.V. Vien), antonio.carcamo@usm.cl (A.E. Cárcamo Hernández), hnlong@iop.vast.ac.vn (H.N. Long).

http://dx.doi.org/10.1016/j.nuclphysb.2016.10.010
0550-3213/

are very close to the SM expectation, there is still room for new extra scalar states. The search of these new scalar states will shed light on the underlying theory behind Electroweak Symmetry Breaking (EWSB) and is the priority of the LHC experiments. Furthermore, despite its great experimental success, the SM has several unaddressed issues, such as, for example, the observed charged fermion mass and quark mixing pattern, the tiny neutrino masses and the sizable leptonic mixing angles, which contrast with the small quark mixing angles. The global fits of the available data from the Daya Bay [3], T2K [4], MINOS [5], Double CHOOZ [6] and RENO [7] neutrino oscillation experiments, provide constraints on the neutrino mass squared splittings and mixing parameters [8]. It is well known that the charged fermion mass hierarchy spans over a range of five orders of magnitude in the quark sector and a much wider range, which includes extra six orders of magnitude, corresponding to the number of orders of magnitude between the neutrino mass scale and the electron mass. The charged fermion masses can be accommodated in the SM, at the price of having an unnatural tuning among its different Yukawa couplings. Furthermore, experiments with solar, atmospheric and reactor neutrinos [3–7,9] provide clear indications of neutrino oscillations, originated by nonvanishing neutrino masses. All these unexplained issues suggest that new physics have to be invoked to address the fermion puzzle of the SM.

The unexplained flavor puzzle of the SM motivates to consider extensions of the SM that explain the fermion mass and mixing pattern. From the phenomenological point of view, one can assume Yukawa textures [10–34] to explain some features of the fermion mass hierarchy. Discrete flavor groups provide a very promising approach to address the flavor puzzle, and been extensively used in several models to explain the prevailing pattern of fermion masses and mixings (see Refs. [35–38] for recent reviews on flavor symmetries). Models with spontaneously broken flavor symmetries may also produce hierarchical mass structures. Recently, discrete groups such as A_4 [39–60], S_3 [61–76], S_4 [77–85], D_4 [86–95], T_7 [96–105], T_{13} [106–109], T' [110–115] and $\Delta(27)$ [116–123] have been implemented in extensions of the SM to explain the prevailing fermion mass and mixing pattern.

Besides that, another unanswered issue in particle physics is the existence of three families of fermions at low energies. The origin of the family structure of the fermions can be addressed in family dependent models where a symmetry distinguish fermions of different families. This issue can be explained by the models based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes U(1)_X$ gauge symmetry, also called 3-3-1 models, which include a family non-universal $U(1)_X$ symmetry [25,58,59,72,73,102,104,124–156]. These models have several phenomenological advantages. Firstly, the three family structure in the fermion sector can be explained in the 3-3-1 models from the chiral anomaly cancellation and asymptotic freedom in QCD [157–159]. Secondly, the fact that the third family is treated under a different representation, can explain the large mass difference between the heaviest quark family and t he two lighter ones. Finally, these models contain a natural Peccei–Quinn symmetry, necessary to solve the strong-CP problem [152]. Furthermore, the 331 models with sterile neutrinos have weakly interacting massive fermionic dark matter candidates [153].

In the 3-3-1 models, the $\mathrm{SU}(3)_L \otimes U(1)_X$ symmetry is broken down to the SM electroweak group $\mathrm{SU}(2)_L \otimes U(1)_Y$ by one heavy $\mathrm{SU}(3)_L$ triplet field that gets a Vacuum Expectation Value (VEV) at high energy scale v_χ, thus giving masses to non-SM fermions and gauge bosons, while the Electroweak Symmetry Breaking is triggered by the remaining lighter triplets as well as by $\mathrm{SU}(3)_L$ antisextets in some version of the model, with VEVs at the electroweak scale v_ρ and v_η, thus providing masses for SM fermions and gauge bosons [25].

In this paper we propose a 3-3-1 model based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathcal{L}} \otimes \Delta(27)$ symmetry consistent with fermion masses and mixings. Our model is the first 331 model based on the $\Delta(27)$ family symmetry, proposed in the literature.[1] Our model also includes a new $\mathrm{U}(1)_{\mathcal{L}}$ that allows us to treat the quark, charged lepton and neutrino sector independently. The light active neutrino masses arise from a combination of type I and type II seesaw mechanisms mediated by three heavy right handed Majorana neutrinos and three $\mathrm{SU}(3)_L$ scalar antisextets, respectively. The content of this paper goes as follows. In Sec. 2 we explain some theoretical aspects of our 331 model. The charged fermion sector is discussed in Sec. 2.1. In Sec. 2.2 we focus on the discussion of the neutrino sector as well as in lepton masses and mixing and give our corresponding results. In Sec. 3, we discuss the implications of our model in the quark sector. Conclusions are given in Sec. 4. In the appendices we present several technical details: Appendices A and B give a detailed description of the $\Delta(27)$ group and the matrices of the 3 representation of $\Delta(27)$, respectively. Appendix C provides the breaking patterns of $\Delta(27)$ by triplets.[2]

2. The model

The symmetry group of the model under consideration is

$$G = \mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathcal{L}} \otimes \Delta(27),$$

where the electroweak factor $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ is extended from those of the SM, and the strong interaction sector is retained. Lets us note that the gauge symmetry of the 331 model is supplemented by the $\mathrm{U}(1)_{\mathcal{L}}$ global and $\Delta(27)$ symmetries. Each lepton family includes a new neutral fermion (N_R) with vanishing lepton number $L(N_R) = 0$ arranged under the $\mathrm{SU}(3)_L$ symmetry as a triplet (ν_L, l_L, N_R^c) and a singlet l_R. The residual electric charge operator Q is therefore related to the generators of the gauge symmetry by [84]

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X,$$

where T_a $(a = 1, 2, ..., 8)$ are $\mathrm{SU}(3)_L$ charges with $\mathrm{Tr} T_a T_b = \frac{1}{2}\delta_{ab}$ and X is the $\mathrm{U}(1)_X$ charge. This means that the model under consideration does not contain exotic electric charges in the fundamental fermion, scalar and adjoint gauge boson representations. Since particles with different lepton number are put in $\mathrm{SU}(3)_L$ triplets, it is better to work with a new conserved charge $\mathcal{L}$ commuting with the gauge symmetry and related to the ordinary lepton number by diagonal matrices [84,160]

$$L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L}.$$

The lepton charge arranged in this way, i.e. $L(N_R) = 0$, is in order to prevent unwanted interactions due to $\mathrm{U}(1)_{\mathcal{L}}$ symmetry and breaking due to the lepton parity to obtain the consistent lepton and quark spectra. By this embedding, exotic quarks U, D as well as new non-Hermitian gauge bosons X^0, $Y^{\pm}$ possess lepton charges as of the ordinary leptons: $L(D) = -L(U) = L(X^0) = L(Y^-) = 1$.

The fermion content and the scalar fields of the model are summarized in Table 1.

[1] In this scenario, only one flavor symmetry $\Delta(27)$ is added.

[2] We prefer to use the notation 3^* for a SU(3) anti-triplet and $\underline{\bar{3}}$ for a $\Delta(27)$ anti-triplet, i.e., all $\Delta(27)$ representations appear with a bar underneath, and the anti-triplets appear also with a bar on top.

Table 1
The fermion content of the model.

Fields	$\psi_{1,2,3L}$	$l_{1,2,3R}$	$Q_{1,2L}$	Q_{3L}	u_R	d_R	U_R	$D_{1,2R}$	ϕ	σ	ρ	η	χ
$\mathrm{SU}(3)_L$	3	1	3*	3	1	1	1	1	3	6*	3	3	3
$\mathrm{U}(1)_X$	$-\frac{1}{3}$	-1	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$\mathrm{U}(1)_{\mathcal{L}}$	$\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	-1	1	$-\frac{1}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
$\underline{\Delta}(27)$	$\underline{3}$	$\underline{1}_1, \underline{1}_2, \underline{1}_3$	$\underline{1}_{1,2}$	$\underline{1}_3$	$\underline{3}$	$\underline{\bar{3}}$	$\underline{1}_2$	$\underline{1}_{1,3}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{\bar{3}}$	$\underline{1}_1$

As we will see in the next sections, the $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_{\mathcal{L}}$ charge assignments for the fermion sector, enforce to have different scalar fields in the quark, charged leptons and neutrino Yukawa interactions. Consequently the $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_{\mathcal{L}}$ symmetries help to treat the charged lepton, neutrino and quark sectors independently.

2.1. Charged-lepton sector

Since left handed $\mathrm{SU}(3)_L$ lepton triplets are unified in a $\Delta(27)$ triplet, to generate charged lepton masses, we need three $\mathrm{SU}(3)_L$ Higgs triplets grouped in a $\underline{3}$ under $\Delta(27)$ given in Table 1. The G assignments of the scalar fields participating in charged lepton Yukawa interactions are:

$$\phi = (\phi_1, \phi_2, \phi_3), \quad \phi_i = \left(\phi_{i1}^+, \phi_{i2}^0, \phi_{i3}^+\right)^T, \quad i = 1, 2, 3. \tag{1}$$

The Yukawa interactions for charged leptons are

$$\begin{aligned} -\mathcal{L}_l &= h_1(\bar{\psi}_L\phi)_{\underline{1}_1} l_{1R} + h_2(\bar{\psi}_L\phi)_{\underline{1}_3} l_{2R} + h_3(\bar{\psi}_L\phi)_{\underline{1}_2} l_{3R} + H.c \\ &= h_1(\bar{\psi}_{1L}\phi_1 + \bar{\psi}_{2L}\phi_2 + \bar{\psi}_{3L}\phi_3)_{\underline{1}_1} l_{1R} \\ &\quad + h_2(\bar{\psi}_{1L}\phi_1 + \omega^2\bar{\psi}_{2L}\phi_2 + \omega\bar{\psi}_{3L}\phi_3)_{\underline{1}_1} l_{2R} \\ &\quad + h_3(\bar{\psi}_{1L}\phi_1 + \omega\bar{\psi}_{2L}\phi_2 + \omega^2\bar{\psi}_{3L}\phi_3)_{\underline{1}_1} l_{3R} + H.c. \end{aligned} \tag{2}$$

To obtain a realistic lepton spectrum, we suppose that in charged lepton sector $\Delta(27)$ is broken down to {Identity}, i.e, it is completely broken. This can be achieved with the VEV alignment $\langle\phi\rangle = (\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_3\rangle)$ under $\Delta(27)$, where $\langle\phi_1\rangle \neq \langle\phi_2\rangle \neq \langle\phi_3\rangle$, and

$$\langle\phi_i\rangle = (0 \quad v_i \quad 0)^T \quad (i = 1, 2, 3). \tag{3}$$

Under this alignment, the mass Lagrangian for the charged leptons reads

$$\mathcal{L}_l^{\mathrm{mass}} = -(\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L}) M_l (l_{1R}, l_{2R}, l_{3R})^T + H.c, \tag{4}$$

where

$$M_l = \begin{pmatrix} h_1 v_1 & h_2 v_1 & h_3 v_2 \\ h_1 v_2 & \omega^2 h_2 v_2 & \omega h_3 v_2 \\ h_1 v_3 & \omega h_2 v_3 & \omega^2 h_3 v_3 \end{pmatrix}. \tag{5}$$

As will be shown in section 2.2, in the case $v_1 = v_2 = v_3 = v$, i.e, $\Delta(27)$ is broken into Z_3 group which consisting of the elements $\{1, b, b^2\}$, the charged lepton matrix M_l in Eq. (5) is diagonalized by the matrix

$$U_{0L} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \tag{6}$$

and the exact tri-bimaximal mixing form will obtained. For a detailed study of this problem, the reader can see Ref. [102].

As we know, the realistic lepton mixing form is a small deviation from tri-bimaximal form [9]. This can be achieved with a small difference between v_2, v_3 and v_1. Therefore we can separate v_2, v_3 into two parts, the first is equal to $v_1 \equiv v$, the second is responsible for the deviation,

$$v_1 = v,\ \ v_2 = v(1+\varepsilon_2),\ \ v_3 = v(1+\varepsilon_3),\ \ \varepsilon_{2,3} \ll 1, \tag{7}$$

and the matrix M_l in (5) becomes

$$\begin{aligned} M_l &= \begin{pmatrix} h_1 v & h_2 v & h_3 v \\ h_1 v(1+\varepsilon_2) & \omega^2 h_2 v(1+\varepsilon_2) & \omega h_3 v(1+\varepsilon_2) \\ h_1 v(1+\varepsilon_3) & \omega h_2 v(1+\varepsilon_3) & \omega^2 h_3 v(1+\varepsilon_3) \end{pmatrix} \\ &\equiv v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+\varepsilon_2 & 0 \\ 0 & 0 & 1+\varepsilon_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{pmatrix}. \end{aligned} \tag{8}$$

The matrix M_l in Eq. (8) can be diagonalized by two steps as follows.

Firstly, we denote

$$M'_l = U^+_{0L} M_l = \frac{v}{\sqrt{3}} \begin{pmatrix} (3+\varepsilon_2+\varepsilon_3)h_1 & (\omega^2\varepsilon_2+\omega\varepsilon_3)h_2 & (\omega\varepsilon_2+\omega^2\varepsilon_3)h_3 \\ (\omega\varepsilon_2+\omega^2\varepsilon_3)h_1 & (3+\varepsilon_2+\varepsilon_3)h_2 & (\omega^2\varepsilon_2+\omega\varepsilon_3)h_3 \\ (\omega^2\varepsilon_2+\omega\varepsilon_3)h_1 & (\omega\varepsilon_2+\omega^2\varepsilon_3)h_2 & (3+\varepsilon_2+\varepsilon_3)h_3 \end{pmatrix}.$$

Secondly, the matrix M'_l in Eq. (9) is diagonalized by

$$U^+_L M'_l \equiv U^+_L U^+_{0L} M_l = \mathrm{diag}(m_e, m_\mu, m_\tau), \tag{9}$$

where

$$m_e = Y_l h_1 v,\ \ m_\mu = Y_l h_2 v,\ \ m_\tau = Y_l h_3 v, \tag{10}$$

with

$$Y_l = \frac{3\sqrt{3}(1+\varepsilon_3)[\varepsilon_3(\varepsilon_3+\varepsilon-4)-4]}{(2+\varepsilon_3)[\varepsilon_3(\varepsilon_3+\varepsilon-6)-6]},$$

and

$$\varepsilon = \sqrt{\varepsilon_3^2 - 12(\varepsilon_3+1)}.$$

The matrix that diagonalize M'_l in (9) takes the form:

$$U_L = \begin{pmatrix} 1 & U^l_{12} & U^l_{13} \\ U^l_{13} & 1 & U^l_{12} \\ U^l_{12} & U^l_{13} & 1 \end{pmatrix}, \quad U_R = 1 \tag{11}$$

where

$$\begin{aligned} U^l_{12} &= \frac{\varepsilon_3\left\{6-2i\sqrt{3}-(1+i\sqrt{3})\varepsilon+\varepsilon_3[7-i\sqrt{3}+(1-i\sqrt{3})(\varepsilon-\varepsilon_3)]\right\}}{2(2+\varepsilon_3)[-6+\varepsilon_3^2-\varepsilon_3(6+\varepsilon)]}, \\ U^l_{13} &= \frac{\varepsilon_3\left\{6+2i\sqrt{3}-(1-i\sqrt{3})\varepsilon+\varepsilon_3[7+i\sqrt{3}+(1+i\sqrt{3})(\varepsilon-\varepsilon_3)]\right\}}{2(2+\varepsilon_3)[-6+\varepsilon_3^2-\varepsilon_3(6+\varepsilon)]}. \end{aligned} \tag{12}$$

To get the results in Eq. (12) we have used the following relations

$$\varepsilon_2 = \frac{\varepsilon_3(2-\varepsilon_3-\varepsilon)}{2(2+\varepsilon_3)}, \quad \varepsilon_2^* = \frac{\varepsilon_3(-2-3\varepsilon_3+\varepsilon)}{2(1+\varepsilon_3)(2+\varepsilon_3)}, \quad \varepsilon_3^* = \frac{1}{1+\varepsilon_3} - 1,$$

which are obtained from the unitary condition of U_L.

The left- and right-handed mixing matrices in charged lepton sector are given by:

$$U'_L = U_{0L}.U_L = \begin{pmatrix} \alpha_1 & \alpha_1 & \alpha_1 \\ \alpha_2 & \omega^2\alpha_2 & \omega\alpha_2 \\ \alpha_3 & \omega\alpha_3 & \omega^2\alpha_3 \end{pmatrix}, \; U'_R = 1, \tag{13}$$

where[3]

$$\alpha_1 = \frac{\sqrt{3}\left[\varepsilon_3^2 - \varepsilon_3(\varepsilon+4) - 4\right]}{(2+\varepsilon_3)[\varepsilon_3^2 - \varepsilon_3(\varepsilon+6) - 6]}, \quad \alpha_2 = \frac{2\sqrt{3}(1+\varepsilon_3)}{6-\varepsilon_3^2+\varepsilon_3(6+\varepsilon)}, \quad \alpha_3 = (1+\varepsilon_3)\alpha_1. \tag{14}$$

In the case $\varepsilon_3 = 0$ it follows that $\varepsilon_2^* = \varepsilon_2 = \varepsilon_3^* = 0$, $U_L = 1$ and the lepton mixing U'_L in Eq. (13) reduces to tri-bimaximal form (U_{HPS}) [161] which is ruled out by the recent data [9]. In general $\varepsilon_{2,3} \neq 0$ (but small) so α_i $(i = 1, 2, 3)$ in Eq. (14) are a little different to each other and different from $\frac{1}{\sqrt{3}}$. Consequently, the lepton mixing U'_L in Eq. (13) differs to U_{HPS} and can lead to the realistic lepton mixing with non-zero θ_{13} as represented in Sec. 2.2. This is one of the striking results of the model under consideration.

Taking into account of the discovery of the long-awaited Higgs boson at around 125 GeV by ATLAS [1] and CMS [2], we can choose[4] $v = 100$ GeV for its scale. From (10), the charged lepton Yukawa couplings $h_{1,2,3}$ relate to their masses as follows:

$$h_1 = m_e/Y_l v, \;\; h_2 = m_\mu/Y_l v, \;\; h_3 = m_\tau/Y_l v. \tag{15}$$

The best fit values for the charged lepton masses are given in Ref. [9]:

$$m_e \simeq 0.511 \text{ MeV}, \;\; m_\mu \simeq 105.66 \text{ MeV}, \;\; m_\tau \simeq 1776.82 \text{ MeV}. \tag{16}$$

With the help of Eqs. (16) and (15) we get $\frac{h_1}{h_2} \simeq 0.0048$, $\frac{h_1}{h_3} \simeq 0.00029$ and $\frac{h_2}{h_3} = 0.0595$, i.e., $h_1 \ll h_2 \ll h_3$ for ε_3 is arbitrary. As will be shown in Sec. 2.2, from the experimental constrains on lepton mixing [162], we obtain a solution in Eq. (30). With this solution, we get

$$h_1 = 2.96671 \times 10^{-6}, \;\; h_2 = 6.13429 \times 10^{-4}, \;\; h_3 = 1.03157 \times 10^{-2}.$$

We note that the mass hierarchy of the charged leptons are well separated by only one Higgs triplet ϕ of $\Delta(27)$, and this is one of the good features of the $\Delta(27)$ group.

2.2. Neutrino masses and mixings

The neutrino masses arise from the coupling of $\bar{\psi}^c_L\psi_L$ to scalars, where $\bar{\psi}^c_L\psi_L$ transforms as $3^* \oplus 6$ under $\mathrm{SU}(3)_L$ and $\underline{3} \oplus \underline{3} \oplus \underline{3}$ under $\Delta(27)$. It is worth noting that under the $\Delta(27)$ group,

[3] With the value of ϵ obtained in Eq. (30), $|\alpha_1| \simeq |\alpha_2| \simeq |\alpha_3| = 0.577 \simeq 1/\sqrt{3}$.

[4] In the SM, the Higgs VEV is equal to 246 GeV, fixed by the W boson mass $m_W^2 = \frac{g^2}{4}v_{weak}^2$, and in the model under consideration, $M_W^2 \simeq \frac{g^2}{2}\left(3u^2 + 3v^2\right)$. Therefore, we can identify $v_{weak}^2 = 6(u^2+v^2) = (246\text{ GeV})^2$ and then obtain $u \sim v \simeq 71$ GeV.

$\underline{3} \otimes \underline{3} \otimes \underline{3}$ has three invariants. Consequently, to build neutrino Yukawa terms invariant under the symmetries of the model, that give rise to light active neutrino masses via type I and type II seesaw mechanisms, we enlarge the scalar sector of the 331 model by introducing three $\mathrm{SU}(3)_L$ scalar antisextets, namely σ_i $(i=1,2,3)$ as well as extra three $\mathrm{SU}(3)_L$ scalar triplets, denoted as ρ_i $(i=1,2,3)$ grouped in $\Delta(27)$ triplets as given in Table 1. The scalar fields participating in the neutrino Yukawa interactions have the following assignments under the $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \otimes \mathrm{U}(1)_{\mathcal{L}} \otimes \Delta(27)$ group:

$$\sigma=(\sigma_1,\sigma_2,\sigma_3), \quad \sigma_i=\begin{pmatrix} \sigma^0_{11} & \sigma^+_{12} & \sigma^0_{13} \\ \sigma^+_{12} & \sigma^{++}_{22} & \sigma^+_{23} \\ \sigma^0_{13} & \sigma^+_{23} & \sigma^0_{33} \end{pmatrix}_i, \quad i=1,2,3, \tag{17}$$
$$\rho=(\rho_1,\rho_2,\rho_3), \quad \rho_i=\begin{pmatrix} \rho^+_{i1} & \rho^0_{i2} & \rho^+_{i3} \end{pmatrix}^T.$$

Furthermore, we assume the following VEV patterns for the $\Delta(27)$ scalar triplets σ and ρ:

$$\langle\sigma\rangle=(\langle\sigma_1\rangle,0,0), \qquad \langle\rho\rangle=(0,0,\langle\rho_3\rangle)$$

where

$$\langle\sigma_1\rangle=\begin{pmatrix} \lambda_\sigma & 0 & v_\sigma \\ 0 & 0 & 0 \\ v_\sigma & 0 & \Lambda_\sigma \end{pmatrix}, \qquad \langle\rho_3\rangle=(0,v_\rho,0)^T,$$

i.e., $\Delta(27)$ is broken into Z_3 groups which consisting of the elements $\{e, aa', (aa')^2\}$ and $\{e, a', a'^2\}$ by σ and ρ, respectively.

The neutrino Yukawa interactions invariant under the symmetries of the model are given by[5]:

$$\begin{aligned}
-\mathcal{L}_\nu &= \frac{x}{2}(\bar{\psi}^c_L\sigma)_{\underline{3}}\psi_L + \frac{y}{2}(\bar{\psi}^c_L\sigma)_{\bar{\underline{3}}}\psi_L + \frac{z}{2}(\bar{\psi}^c_L\rho)_{\bar{\underline{3}}}\psi_L + H.c \\
&= \frac{x}{2}(\bar{\psi}^c_{1L}\sigma_1\psi_{1L} + \bar{\psi}^c_{2L}\sigma_2\psi_{2L} + \bar{\psi}^c_{3L}\sigma_3\psi_{3L}) \\
&\quad + \frac{y}{2}\big(\bar{\psi}^c_{2L}\sigma_3\psi_{1L} + \bar{\psi}^c_{3L}\sigma_2\psi_{1L} + \bar{\psi}^c_{3L}\sigma_1\psi_{2L} \\
&\quad + \bar{\psi}^c_{1L}\sigma_3\psi_{2L} + \bar{\psi}^c_{1L}\sigma_2\psi_{3L} + \bar{\psi}^c_{2L}\sigma_1\psi_{3L}\big) \\
&\quad + \frac{z}{2}\big(\bar{\psi}^c_{2L}\rho_3\psi_{1L} - \bar{\psi}^c_{3L}\rho_2\psi_{1L} + \bar{\psi}^c_{3L}\rho_1\psi_{2L} \\
&\quad - \bar{\psi}^c_{1L}\rho_3\psi_{2L} + \bar{\psi}^c_{1L}\rho_2\psi_{3L} - \bar{\psi}^c_{2L}\rho_1\psi_{3L}\big) + H.c.
\end{aligned} \tag{18}$$

Then, it follows that the neutrino mass terms are

$$\begin{aligned}
-\mathcal{L}^{mass}_\nu &= \frac{1}{2}x[\lambda_\sigma\bar{\nu}^c_{1L}\nu_{1L} + v_\sigma\bar{N}_{1R}\nu_{1L} + v_\sigma\bar{\nu}^c_{1L}N^c_{1R} + \Lambda_\sigma\bar{N}_{1R}N^c_{1R}] \\
&\quad + \frac{1}{2}y\big[\lambda_\sigma\bar{\nu}^c_{2L}\nu_{3L} + v_\sigma\bar{N}_{2R}\nu_{3L} + v_\sigma\bar{\nu}^c_{2L}N^c_{3R} + \Lambda_\sigma\bar{N}_{2R}N^c_{3R}
\end{aligned}$$

---

[5] The following terms are invariant under the symmetries of the model: $(\bar{\psi}^c_L\sigma)_{\bar{\underline{3}}}\psi_L = \bar{\psi}^c_{2L}\sigma_3\psi_{1L} - \bar{\psi}^c_{3L}\sigma_2\psi_{1L} + \bar{\psi}^c_{3L}\sigma_1\psi_{2L} - \bar{\psi}^c_{1L}\sigma_3\psi_{2L} + \bar{\psi}^c_{1L}\sigma_2\psi_{3L} - \bar{\psi}^c_{2L}\sigma_1\psi_{3L}$, $(\bar{\psi}^c_L\rho)_{\underline{3}}\psi_L = \bar{\psi}^c_{1L}\rho_1\psi_{1L} + \bar{\psi}^c_{2L}\rho_2\psi_{2L} + \bar{\psi}^c_{3L}\rho_3\psi_{3L}$, and $(\bar{\psi}^c_L\rho)_{\bar{\underline{3}}}\psi_L = \bar{\psi}^c_{2L}\rho_3\psi_{1L} + \bar{\psi}^c_{3L}\rho_2\psi_{1L} + \bar{\psi}^c_{3L}\rho_1\psi_{2L} + \bar{\psi}^c_{1L}\rho_3\psi_{2L} + \bar{\psi}^c_{1L}\rho_2\psi_{3L} + \bar{\psi}^c_{2L}\rho_1\psi_{3L}$ but they are all vanish, i.e., they have no contribution to the neutrino mass matrices $M_{L,D,R}$.

$$+ \lambda_\sigma \bar{\nu}^c_{3L}\nu_{2L} + v_\sigma \bar{N}_{3R}\nu_{2L} + v_\sigma \bar{\nu}^c_{3L} N^c_{2R} + \Lambda_\sigma \bar{N}_{3R} N^c_{2R}\big]$$
$$+ \frac{1}{2} z \left[v_\rho \bar{\nu}^c_{2L} N^c_{1R} - v_\rho \bar{N}_{2R}\nu_{1L} - v_\rho \bar{\nu}^c_{1L} N^c_{2R} + v_\rho \bar{N}_{1R}\nu_{2L}\right] + H.c. \tag{19}$$

We can rewrite (19) in the matrix form

$$-\mathcal{L}^{\text{mass}}_\nu = \frac{1}{2}\bar{\chi}^c_L M_\nu \chi_L + H.c., \quad \chi_L \equiv \begin{pmatrix} \nu_L \\ N^c_R \end{pmatrix}, \quad M_\nu \equiv \begin{pmatrix} M_L & M^T_D \\ M_D & M_R \end{pmatrix}, \tag{20}$$

where $\nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T$, $N_R = (N_{1R}, N_{2R}, N_{3R})^T$ and

$$M_{L,D,R} = \begin{pmatrix} a_{L,D,R} & c_{L,D,R} & 0 \\ -c_{L,D,R} & 0 & b_{L,D,R} \\ 0 & b_{L,D,R} & 0 \end{pmatrix}, \tag{21}$$

with

$$\begin{aligned} &a_L = \lambda_\sigma x, \quad a_D = v_\sigma x, \quad a_R = \Lambda_\sigma x, \\ &b_L = \lambda_\sigma y, \quad b_D = v_\sigma y, \quad b_R = \Lambda_\sigma y, \\ &c_L = 0, \quad c_D = v_\rho z, \quad c_R = 0. \end{aligned} \tag{22}$$

The effective neutrino mass matrix, in the framework of type I and type II seesaw mechanisms, is given by[6]

$$\begin{aligned} M_{\text{eff}} &= M_L - M^T_D M^{-1}_R M_D \\ &= \begin{pmatrix} a_L - \frac{a^2_D}{a_R} & 0 & 0 \\ 0 & -\frac{c^2_D}{a_R} & b_L - \frac{b^2_D}{b_R} \\ 0 & b_L - \frac{b^2_D}{b_R} & 0 \end{pmatrix} \equiv \begin{pmatrix} A & 0 & 0 \\ 0 & C & B \\ 0 & B & 0 \end{pmatrix}, \end{aligned} \tag{23}$$

where

$$A = a_L - \frac{a^2_D}{a_R}, \quad B = b_L - \frac{b^2_D}{b_R}, \quad C = -\frac{c^2_D}{a_R}.$$

In the case without the ρ contribution ($v_\rho = 0$) we have $c_D = 0$ and M_{eff} in (23) becomes

$$M^0_{\text{eff}} = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & B \\ 0 & B & 0 \end{pmatrix}. \tag{24}$$

The mass matrix in Eq. (24) gives the degenerate mass of neutrinos

$$m^0_1 = -m^0_3 = B, \ \ m^0_2 = A,$$

and the corresponding leptonic mixing matrix yields the tri-bimaximal mixing form $U^+_L U_\nu = U_{HPS}$, which is ruled out by the recent neutrino experimental data. However, the ρ contribution will improve this. Indeed, the mass matrix (23) is diagonalized as follows $U^T_\nu M_{\text{eff}} U_\nu = \text{diag}(m_1, m_2, m_3)$, with

[6] With $a_{D,R}, b_{D,R}$ given in Eq. (22), $\frac{b_D}{b_R} - \frac{a_D}{a_R} = 0$, and $(M_{\text{eff}})_{12} = (M_{\text{eff}})_{21} = \left(\frac{b_D}{b_R} - \frac{a_D}{a_R}\right) c_D = 0$.

$$m_{1,3} = \frac{1}{2}\left(C \pm \sqrt{C^2 + 4B^2}\right), \quad m_2 = A, \tag{25}$$

and the corresponding neutrino mixing matrix:

$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ \frac{K}{\sqrt{K^2+1}} & 0 & -\frac{1}{\sqrt{K^2+1}} \\ \frac{1}{\sqrt{K^2+1}} & 0 & \frac{K}{\sqrt{K^2+1}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{26}$$

where

$$K = \frac{C - \sqrt{C^2 + 4B^2}}{2B}. \tag{27}$$

Combining (13) and (26), the lepton mixing matrix takes the form:

$$U_{Lep} = U_L'^{+} U_\nu = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{28}$$

where

$$\begin{aligned} u_{11} &= \frac{K\beta_2 + \beta_3}{\sqrt{K^2+1}}, \quad u_{12} = u_{22} = u_{32} = \beta_1, \\ u_{13} &= \frac{-\beta_2 + K\beta_3}{\sqrt{K^2+1}}, \quad u_{21} = \frac{\omega(K\beta_2 + \omega\beta_3)}{\sqrt{K^2+1}}, \quad u_{23} = \frac{\omega(-\beta_2 + K\omega\beta_3)}{\sqrt{K^2+1}}, \\ u_{31} &= \frac{\omega(K\omega\beta_2 + \beta_3)}{\sqrt{K^2+1}}, \quad u_{33} = \frac{\omega(-\omega\beta_2 + K\beta_3)}{\sqrt{K^2+1}}, \end{aligned} \tag{29}$$

with

$$\beta_i = \frac{1}{3\alpha_i} \ (i = 1, 2, 3).$$

We see that all the elements of the matrix U_{Lep} in Eq. (29) depend only on two parameters ε_3 ans K. From experimental constraints on the elements of the lepton mixing matrix given in Refs. [162–164], we can find out the regions of K and ε_3 that satisfy experimental data on lepton mixing matrix. Indeed, in the case $\alpha_i = \beta_i = 1/\sqrt{3}$ $(i = 1, 2, 3)$ and $K = 1$, the lepton mixing matrix in Eq. (28) reduces to tri-bimaximal form. Therefore, the realistic lepton mixing pattern can be obtained if the values of α_i, β_i $(i = 1, 2, 3)$ are close to $1/\sqrt{3}$ and K gets values close to unity. If $\alpha_i = \beta_i = 1/\sqrt{3}$ $(i = 1, 2, 3)$, the element u_{11} in Eq. (29) becomes, $u_{11} = \frac{K+1}{\sqrt{3(K^2+1)}}$. By using the experimental constraint values of u_{11} given in [162–164], $0.801 \leq |u_{11}| \leq 0.845$ we get $1.1 \leq |K| \leq 1.5$ which is depicted in Fig. 1.

To get the specific value of ε_3, a specific value of K would be chosen with an experimental value of u_{11}. In the case $K = \sqrt{2} \simeq 1.4142$, combining with the constraint values on the element u_{11} of lepton mixing matrix [162–164], $u_{11} = 0.805$, we obtain a solution[7]:

$$\varepsilon_3 = -0.000743889 + 0.000785038i. \tag{30}$$

[7] In this model, the choice of the parameters is not unique. It is just one specific example to show that there exist the model parameters consistent with the experimental data.

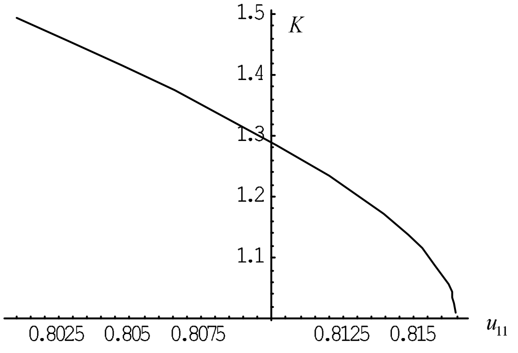

Fig. 1. K as a function of u_{11} with $u_{11} \in (0.801, 0.845)$ [162–164].

Then, it follows that the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix takes the form:

$$U_{Lep} \simeq \begin{pmatrix} 0.805 & 0.577 & 0.137988i \\ -0.402851 + 0.119716i & 0.577 & 0.696899 - 0.0691182i \\ -0.402149 - 0.119716i & 0.577 & -0.697328 - 0.0688701i \end{pmatrix}, \tag{31}$$

which implies that

$$|U_{Lep}| = \begin{pmatrix} 0.805 & 0.577 & 0.137988 \\ 0.420263 & 0.577 & 0.70031 \\ 0.41959 & 0.577 & 0.70072 \end{pmatrix}. \tag{32}$$

Using Eq. (27) and $K = \sqrt{2}$, we obtain

$$C = \frac{B}{\sqrt{2}}. \tag{33}$$

In the standard Particle Data Group (PDG) parametrization, the lepton mixing matrix can be parametrized as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathcal{P}, \tag{34}$$

where $\mathcal{P} = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$, and $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with θ_{12}, θ_{23} and θ_{13} being the solar, atmospheric and reactor angles, respectively. $\delta = [0, 2\pi]$ is the Dirac CP violation phase while α and β are two Majorana CP violation phases. The observable angles in the standard PMNS parametrization are given by [9]

$$s_{13} = |U_{13}|, \; s_{23} = \frac{|U_{23}|}{\sqrt{1 - |U_{13}|^2}}, \; s_{12} = \frac{|U_{12}|}{\sqrt{1 - |U_{13}|^2}}. \tag{35}$$

Combining Eqs. (32) and (35) yields:

$$\sin\theta_{13} = 0.137988, \; \sin\theta_{23} = 0.713911, \; \sin\theta_{12} = 0.588205, \tag{36}$$

or

$$\theta_{13} \simeq 7.9315^\circ, \; \theta_{23} \simeq 45.5541^\circ, \; \theta_{12} \simeq 36.0293^\circ, \tag{37}$$

which are all very consistent with the recent data on neutrino mixing angles. Furthermore, comparing the lepton mixing matrix given in Eq. (31) with the standard parametrization in Eq. (34), one obtains vanishing Majorana phases, i.e., $\alpha = 0$, $\beta = 0$ as well as nonvanishing leptonic Dirac CP violating phase $\delta = -\frac{\pi}{2}$ and Jarskog invariant close to -3.2×10^{-2}. It is worth mentioning that having leptonic mixing parameters consistent with their experimental values, require that the parameter K to be equal or very close to $\sqrt{2}$. The other parameters that determine the leptonic mixing angles are $Re\,(\varepsilon_3)$ and $Im\,(\varepsilon_3)$, i.e., which are of the order of 10^{-4}. Besides that we have numerically checked the leptonic mixing parameters have a low sensitivity with $Re\,(\varepsilon_3)$ and $Im\,(\varepsilon_3)$ but are highly sensitive under small variations around $K = \sqrt{2}$, for example having $K = 0.9\sqrt{2} \simeq 1.27$ leads to $\sin^2\theta_{13} = 0.009$, which is outside the 3σ experimentally allowed range. In the region of parameter space consistent with the experimental values of the leptonic mixing parameters, we have numerically checked that the leptonic Dirac CP violating phase is equal to $-\frac{\pi}{2}$. Other phases different than $-\frac{\pi}{2}$ are obtained for values of the K parameters outside the vicinity of $K = \sqrt{2}$, that leads to a reactor mixing angle θ_{13} unacceptably small.

At present, the absolute neutrino masses as well as the mass ordering of neutrinos is unknown. The result in [165] shows that

$$m_i \leq 0.6 \text{ eV}, \qquad i = 1, 2, 3, \tag{38}$$

while the upper bound on the sum of light active neutrino masses is given by [166]

$$\sum_{i=1}^{3} m_i \leq 0.5 \text{ eV}. \tag{39}$$

The neutrino mass spectrum can be described by the normal mass hierarchy ($|m_1| \simeq |m_2| < |m_3|$), the inverted hierarchy ($|m_3| < |m_1| \simeq |m_2|$) or the nearly degenerate ($|m_1| \simeq |m_2| \simeq |m_3|$) ordering. The neutrino mass ordering depends on the sign of Δm^2_{23}, which is currently unknown. In the case of 3-neutrino mixing, in the model under consideration, the two possible signs of Δm^2_{23} correspond to two types of allowed neutrino mass spectra.

2.3. *Normal case ($\Delta m^2_{23} > 0$)*

Substituting B from (33) into (25) and taking into account the experimental values of the neutrino mass squared splittings for the normal hierarchy given in [9], i.e., $\Delta m^2_{21} = 7.53 \times 10^{-5}$ eV^2, $\Delta m^2_{32} = 2.44 \times 10^{-3}$ eV^2, we get the following solution:

$$A = 0.030228, \ B = 0.0409496, \ C = 0.0289557, \tag{40}$$

which implies that:

$$|m_1| = 0.0289557 \text{ eV}, \ m_2 = 0.030228 \text{ eV}, \ m_3 = 0.0579114 \text{ eV}. \tag{41}$$

$$x = \frac{0.030228\Lambda_\sigma}{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}, \quad y = \frac{0.0409496\Lambda_\sigma}{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}, \quad z = \frac{0.029585i\Lambda_\sigma}{v_\rho\sqrt{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}}. \tag{42}$$

2.4. *Inverted case ($\Delta m^2_{23} < 0$)*

Substituting B from (33) into (25) and taking into account the neutrino oscillation experimental data of neutrino mass squared differences for the inverted neutrino mass orderings given in [9], i.e., $\Delta m^2_{21} = 7.53 \times 10^{-5}$ eV^2, $\Delta m^2_{32} = 2.52 \times 10^{-3}$ eV^2, we find the solution:

$$A = 0.0577486, \ B = -0.0403708, \ C = -0.0285465, \tag{43}$$

which implies that:

$$|m_1| = 0.0570929 \text{ eV}, \ m_2 = 0.0577486 \text{ eV}, \ m_3 = 0.0285465 \text{ eV}. \tag{44}$$

$$x = \frac{0.0577486\Lambda_\sigma}{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}, \quad y = \frac{0.0403708\Lambda_\sigma}{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}, \quad z = \frac{0.0577486 i \Lambda_\sigma}{v_\rho\sqrt{\Lambda_\sigma\lambda_\sigma - v_\sigma^2}}. \tag{45}$$

2.5. *Effective Majorana neutrino mass parameter*

In what follows we proceed to compute the effective Majorana neutrino mass parameter, whose value is proportional to the amplitude of neutrinoless double beta ($0\nu\beta\beta$) decay. The effective Majorana neutrino mass parameter has the form:

$$m_{\beta\beta} = \left|\sum_j U_{ek}^2 m_{\nu_k}\right|, \tag{46}$$

where U_{ej}^2 is the squared of the PMNS leptonic mixing matrix elements and m_{ν_k} correspond to the masses of the Majorana neutrinos.

From Eqs. (41), (44), (31) and (46), it follows that the effective Majorana neutrino mass parameter, for the Normal and Inverted neutrino mass orderings, acquires the following values:

$$m_{\beta\beta} = \begin{cases} 10 \text{ meV} & \text{for} \quad \text{Normal Hierarchy} \\ 18 \text{ meV} & \text{for} \quad \text{Inverted Hierarchy} \end{cases} \tag{47}$$

As seen from Eq. (47), the resulting effective Majorana neutrino mass parameters for normal and inverted neutrino mass orderings, are out the scope of the present and future $0\nu\beta\beta$ decay experiments. Let us note that the Majorana neutrino mass parameter has the upper limit $m_{\beta\beta} \leq 160$ meV, corresponding to $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.6 \times 10^{25}$ yr at 90% C.L., as follows from the EXO-200 experiment [167]. That limit is expected to be updated in a not too distant future. The GERDA "phase-II" experiment [168,169] is expected to reach $T_{1/2}^{0\nu\beta\beta}(^{76}\text{Ge}) \geq 2 \times 10^{26}$ yr, corresponding to $m_{\beta\beta} \leq 100$ meV. A bolometric CUORE experiment, using ^{130}Te [170], is currently under construction and its estimated sensitivity is about $T_{1/2}^{0\nu\beta\beta}(^{130}\text{Te}) \sim 10^{26}$ yr, corresponding to $m_{\beta\beta} \leq 50$ meV. Besides that, there are plans for ton-scale next-to-next generation $0\nu\beta\beta$ experiments with ^{136}Xe [171,172] and ^{76}Ge [168,173] asserting sensitivities over $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr, corresponding to $m_{\beta\beta} \sim 12$–30 meV. A review on the theory and phenomenology of neutrinoless double-beta decay can be found in Ref. [174]. It is worth mentioning that our model predicts $T_{1/2}^{0\nu\beta\beta}$ at the level of sensitivities of the next generation or next-to-next generation $0\nu\beta\beta$ experiments.

3. Quark masses

The $[\text{SU}(3)_L, \text{U}(1)_X, \text{U}(1)_\mathcal{L}, \underline{\Delta}(27)]$ assignments for the quark sector of the model are given in Table 1. Thus, in order to generate quark masses, we additionally introduce four extra $\text{SU}(3)_L$ scalar triplets, assigned as a $\Delta(27)$ anti-triplet (η) and a $\Delta(27)$ non-trivial singlet (χ). The scalar fields participating in the quark Yukawa interactions:

$$\eta=(\eta_1,\eta_2,\eta_3), \quad \eta_i=\left(\eta^0_{i1}, \eta^-_{i2}, \eta^0_{i3}\right)^T, \quad i=1,2,3, \tag{48}$$

$$\chi=\left(\chi^0_1, \chi^-_2, \chi^0_3\right)^T, \tag{49}$$

where their G assignments are reported in Table 1 and the VEV pattern of the $\Delta(27)$ triplet η is given as

$$\langle\eta\rangle=(\langle\eta_1\rangle,\langle\eta_2\rangle,\langle\eta_3\rangle)^T, \tag{50}$$

with

$$\langle\eta_i\rangle=\begin{pmatrix} u_i\\ 0\\ 0\end{pmatrix} \ (i=1,2,3), \quad \langle\chi\rangle=\begin{pmatrix}0\\0\\v_\chi\end{pmatrix}.$$

The quark Yukawa interactions are

$$\begin{aligned}-\mathcal{L}_q &= h^d_3\bar{Q}_{3L}(\phi d_R)_{1_2}+h^u_1\bar{Q}_{1L}(\phi^* u_R)_{1_1}+h^u_2\bar{Q}_{2L}(\phi^* u_R)_{1_3}\\ &+h^u_3\bar{Q}_{3L}(\eta u_R)_{1_2}+h^d_1\bar{Q}_{1L}(\eta^* d_R)_{1_1}+h^d_2\bar{Q}_{2L}(\eta^* d_R)_{1_3}\\ &+f_3\bar{Q}_{3L}\chi U_R+f_1\bar{Q}_{1L}\chi^* D_{1R}+f_2\bar{Q}_{2L}\chi^* D_{2R}+H.c.\end{aligned} \tag{51}$$

Then, it follows that the quark mass terms take the form

$$\begin{aligned}-\mathcal{L}^{mass}_q &= -h^u_1 v^*_1\bar{u}_{1L}u_{1R}-h^u_1 v^*_2\bar{u}_{1L}u_{2R}-h^u_1 v^*_3\bar{u}_{1L}u_{3R}\\ &-h^u_2 v^*_1\bar{u}_{2L}u_{1R}-\omega^2 h^u_2 v^*_2\bar{u}_{2L}u_{2R}-\omega h^u_2 v^*_3\bar{u}_{2L}u_{3R}\\ &+h^u_3 u_1\bar{u}_{3L}u_{1R}+\omega h^u_3 u_2\bar{u}_{3L}u_{2R}+\omega^2 h^u_3 u_3\bar{u}_{3L}u_{3R}\\ &+h^d_1 u^*_1\bar{d}_{1L}d_{1R}+h^d_1 u^*_2\bar{d}_{1L}d_{2R}+h^d_1 u^*_3\bar{d}_{1L}d_{3R}\\ &+h^d_2 u^*_1\bar{d}_{2L}d_{1R}+\omega^2 h^d_2 u^*_2\bar{d}_{2L}d_{2R}+\omega h^d_2 u^*_3\bar{d}_{2L}d_{3R}\\ &+h^d_3 v_1\bar{d}_{3L}d_{1R}+\omega h^d_3 v_2\bar{d}_{3L}d_{2R}+\omega^2 h^d_3 v_3\bar{d}_{3L}d_{3R}\\ &+f_3 v_\chi\bar{U}_L U_R+f_1 v^*_\chi\bar{D}_{1L}D_{1R}+f_2 v^*_\chi\bar{D}_{2L}D_{2R}+H.c.\end{aligned} \tag{52}$$

Consequently, the exotic quarks do not mix with the SM quarks. From the quark mass terms given above, it follows that the exotic quark masses are

$$m_U=|f_3 v_\chi|, \quad m_{D_{1,2}}=|f_{1,2}v^*_\chi|,$$

and the SM up-type and down-type quark mass matrices take the form:

$$M_u=\begin{pmatrix}-h^u_1 v^*_1 & -h^u_1 v^*_2 & -h^u_1 v^*_3\\ -h^u_2 v^*_1 & -h^u_2 v^*_2\omega^2 & -h^u_2 v^*_3\omega\\ h^u_3 u_1 & h^u_3 u_2\omega & h^u_3 u_3\omega^2\end{pmatrix}, \quad M_d=\begin{pmatrix}h^d_1 u^*_1 & h^d_1 u^*_2 & h^d_1 u^*_3\\ h^d_2 u^*_1 & h^d_2 u^*_2\omega^2 & h^d_2 u^*_3\omega\\ h^d_3 v_1 & h^d_3 v_2\omega & h^d_3 v_3\omega^2\end{pmatrix}. \tag{53}$$

In the quark sector, we assume that the $\Delta(27)$ discrete group is broken down to the Z_3 subgroup, which consists of the elements $\{1,b,b^2\}$. This breaking is triggered by the $\Delta(27)$ scalar triplet η, with the VEV alignment described in Eq. (50). In the case $v_1=v_2=v_3$, $u_1=u_2=u_3$ and $v^*_i=v_i$, $u^*_i=u_i$ $(i=1,2,3)$, the matrices M_u and M_d given by Eq. (53) are diagonalized by the unitary matrices

$$V^u_{0R}=V^d_{0R}=\frac{1}{3}\begin{pmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{pmatrix}, \quad V^u_L=V^d_L=1,$$

and the quark mixing matrix $V_{\rm CKM} = V_L^{d\dagger} V_L^u = 1$, which is acceptable since the quark mixing matrix is very close to the identity matrix [9]. By an appropriate choice of parameters in the SM quark mass matrices given by Eq. (53), we can successfully reproduce the experimental values of quark masses and quark mixing angles. Furthermore it is noteworthy to mention that our model is an extension of the 3-3-1 model considered in [175]. As pointed out in Refs. [175], the flavor constraints can be fulfilled by considering the scale of breaking of the $\mathrm{SU}(3)_L \otimes U(1)_X$ gauge symmetry much larger than the electroweak symmetry breaking scale $v = 246$ GeV, which corresponds to the alignment limit of the mass matrix for the CP-even Higgs bosons. Consequently, following [175], we expect that the FCNC effects as well as the constraints arising from $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixings will be fulfilled in our model, by considering the scale of breaking of the $\mathrm{SU}(3)_L \otimes U(1)_X$ gauge symmetry much larger than scale of breaking of the electroweak symmetry. In that alignment limit, our model effectively becomes a nine Higgs doublet model, whose scalar sector includes 9 CP even neutral Higgses, 8 CP odd neutral Higgses and 16 charged Higgses. That scalar sector is not predictive as its corresponding scalar potential has many free uncorrelated parameters that can be adjusted to get the required pattern of scalar masses. Therefore, the loop effects of the heavy scalars contributing to certain observables can be suppressed by the appropriate choice of the free parameters in the scalar potential. Fortunately, all these adjustments do not affect the charged fermion and neutrino sector, which is completely controlled by the fermion–Higgs Yukawa couplings. In addition, in models with discrete flavor symmetries, like ours, the deviation of the CKM matrix from the identity can be given by the FCNC effects with the left-handed quarks, but in the alignment limit previously described, such deviations are highly suppressed by the mass of the extra quarks [84].

4. Conclusions

We constructed the first $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes U(1)_X$ model based on the $\Delta(27)$ flavor symmetry supplemented by the $\mathrm{U}(1)_{\mathcal{L}}$ new lepton global symmetry. This $\mathrm{U}(1)_{\mathcal{L}}$ new lepton global symmetry allows us to have different scalar fields in the Yukawa interactions for charged lepton, neutrino and quark sectors, thus allowing us to treat these sectors independently. Our model successfully accounts for fermion masses and mixings. In our model, the neutrino Yukawa interactions include three $\mathrm{SU}(3)_L$ scalar triplets as well as three $\mathrm{SU}(3)_L$ scalar antisextets that allow to implement type II and type I seesaw mechanisms, respectively, for the generation of the light active neutrino masses. Consequently, light active neutrino masses arise from a combination of type-I and type-II seesaw mechanisms, mediated by three heavy right handed Majorana neutrinos and three $\mathrm{SU}(3)_L$ scalar antisextets, respectively. Furthermore, from the consistency of the leptonic mixing angles with their experimental values we obtain a non-vanishing leptonic Dirac CP violating phase equal to $-\frac{\pi}{2}$. In addition, our model features an effective Majorana neutrino mass parameter of neutrinoless double beta decay, with values $m_{\beta\beta} = 10$ and 18 meV for the normal and the inverted neutrino mass hierarchies, respectively.

Acknowledgements

This research has received funding from the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2015.33. A.E.C.H. was supported by DGIP internal Grant No. 111458 and by Proyecto Basal FB0821. H.N. Long thanks Universidad Técnica Federico Santa María for hospitality, where this work was finished. The

Table 2
Character table of $\Delta(27)$.

Class	n	h	$\underline{1}_1$	$\underline{1}_2$	$\underline{1}_3$	$\underline{1}_4$	$\underline{1}_5$	$\underline{1}_6$	$\underline{1}_7$	$\underline{1}_8$	$\underline{1}_9$	$\underline{3}$	$\underline{\bar{3}}$
C_1	1	1	1	1	1	1	1	1	1	1	1	3	3
C_2	1	3	1	1	1	1	1	1	1	1	1	3ω	$3\omega^2$
C_3	1	3	1	1	1	1	1	1	1	1	1	$3\omega^2$	3ω
C_4	3	3	1	ω	ω^2	1	ω^2	ω	1	ω	ω^2	0	0
C_5	3	3	1	ω^2	ω	1	ω	ω^2	1	ω^2	ω	0	0
C_6	3	3	1	1	1	ω^2	ω^2	ω^2	ω	ω	ω	0	0
C_7	3	3	1	ω	ω^2	ω^2	ω	1	ω	ω^2	1	0	0
C_8	3	3	1	ω^2	ω	ω^2	1	ω	ω	1	ω^2	0	0
C_9	3	3	1	1	1	ω	ω	ω	ω^2	ω^2	ω^2	0	0
C_{10}	3	3	1	ω^2	ω	ω	ω^2	1	ω^2	ω	1	0	0
C_{11}	3	3	1	ω	ω^2	ω	1	ω^2	ω^2	1	ω	0	0

visit of H.N. Long to Universidad Técnica Federico Santa María was supported by DGIP internal Grant No. 111458.

Appendix A. $\Delta(27)$ group and Clebsch–Gordan coefficients

The $\Delta(27)$ discrete group is a subgroup of SU(3) and is isomorphic to the semi-direct product group $(Z'_3 \times Z''_3) \rtimes Z_3$. It is also a simple group[8] of the type $\Delta(3n^2)$ with $n = 3$. The $\Delta(27)$ discrete group has 27 elements divided into 11 conjugacy classes, so it has 11 irreducible representations, including two triplets ($\underline{3}$ and its conjugate $\underline{\bar{3}}$) and 9 singlets $\underline{1}_i$ $(i = 1, 2, ..., 9)$. Any element of $\Delta(27)$ can be written as a multiplication of three generators, i.e., b, a and a', in the form $b^k a^m a'^n$, satisfying the relations

$$\begin{aligned} &a^3 = a'^3 = b^3 = 1, \quad aa' = a'a, \\ &bab^{-1} = (a'a)^{-1}, \ ba'b^{-1} = a, \end{aligned} \tag{A.1}$$

where b is a generator of Z_3, and a, a' belong to Z'_3 and Z''_3, respectively.

The character table of $\Delta(27)$ is given in Table 2, where n is the number of elements, h is the order of each element, and $\omega = e^{\frac{2\pi i}{3}} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ is the cube root of unity, obeying $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. The conjugacy classes generated from b, a and a' are presented in Eq. (A.2).

$$\begin{aligned} &C_1 : \{e\}, && h = 1, \\ &C_2 : \{a^2 a'\}, && h = 3, \\ &C_3 : \{aa'^2\}, && h = 3, \\ &C_4 : \{b,\ ba^2a',\ baa'^2\}, && h = 3, \\ &C_5 : \{b^2,\ b^2a^2a'^2aa'^2\}, && h = 3, \\ &C_6 : \{aa'^2,\ a'^2\}, && h = 3, \\ &C_7 : \{ba^2,\ ba'^2,\ baa'\}, && h = 3, \\ &C_8 : \{b^2a'^2,\ b^2aa'^2a^2\}, && h = 3, \end{aligned}$$

[8] In fact, the simplest group of the type $\Delta(3n^2)$ is $\Delta(3) \equiv Z_3$. The next group, $\Delta(12)$, is isomorphic to A_4. Thus, the simplest non-trivial group of the type $\Delta(3n^2)$ is $\Delta(27)$.

Table 3
The singlet multiplications of the group $\Delta(27)$.

Singlets	$\underline{1}_2$	$\underline{1}_3$	$\underline{1}_4$	$\underline{1}_5$	$\underline{1}_6$	$\underline{1}_7$	$\underline{1}_8$	$\underline{1}_9$
$\underline{1}_2$	$\underline{1}_3$	$\underline{1}_1$	$\underline{1}_6$	$\underline{1}_4$	$\underline{1}_5$	$\underline{1}_8$	$\underline{1}_9$	$\underline{1}_7$
$\underline{1}_3$	$\underline{1}_1$	$\underline{1}_2$	$\underline{1}_5$	$\underline{1}_6$	$\underline{1}_4$	$\underline{1}_9$	$\underline{1}_7$	$\underline{1}_8$
$\underline{1}_4$	$\underline{1}_6$	$\underline{1}_5$	$\underline{1}_7$	$\underline{1}_9$	$\underline{1}_8$	$\underline{1}_1$	$\underline{1}_2$	$\underline{1}_3$
$\underline{1}_5$	$\underline{1}_4$	$\underline{1}_6$	$\underline{1}_9$	$\underline{1}_8$	$\underline{1}_7$	$\underline{1}_3$	$\underline{1}_1$	$\underline{1}_2$
$\underline{1}_6$	$\underline{1}_5$	$\underline{1}_4$	$\underline{1}_8$	$\underline{1}_7$	$\underline{1}_9$	$\underline{1}_2$	$\underline{1}_3$	$\underline{1}_1$
$\underline{1}_7$	$\underline{1}_8$	$\underline{1}_9$	$\underline{1}_1$	$\underline{1}_3$	$\underline{1}_2$	$\underline{1}_4$	$\underline{1}_6$	$\underline{1}_5$
$\underline{1}_8$	$\underline{1}_9$	$\underline{1}_7$	$\underline{1}_2$	$\underline{1}_1$	$\underline{1}_3$	$\underline{1}_6$	$\underline{1}_5$	$\underline{1}_4$
$\underline{1}_9$	$\underline{1}_7$	$\underline{1}_8$	$\underline{1}_3$	$\underline{1}_2$	$\underline{1}_1$	$\underline{1}_5$	$\underline{1}_4$	$\underline{1}_6$

$$\begin{aligned}
&C_9:\{a^2a'^2,\ a,\ a'\}, && h=3,\\
&C_{10}:\{ba,\ ba',\ ba^{-1}a'^2\}, && h=3,\\
&C_{11}:\ b^2a',\ b^2a^{-1}a'^2,\ b^2a\}, && h=3.
\end{aligned} \tag{A.2}$$

The multiplication rules for $\Delta(27)$ group are

$$\begin{aligned}
\underline{3}\otimes\underline{3} = \underline{\bar{3}}(x_1y_1,\ x_2y_2,\ x_3y_3) \oplus \underline{\bar{3}}(x_2y_3+x_3y_2,\ x_3y_1+x_1y_3,\ x_1y_2+x_2y_1)\\
\oplus\,\underline{\bar{3}}\,(x_2y_3-x_3y_2,\ x_3y_1-x_1y_3,\ x_1y_2-x_2y_1)\,,
\end{aligned} \tag{A.3}$$

and

$$\underline{3}\otimes\underline{\bar{3}} = \sum_{i=1}^{9}\oplus\underline{1}_i\,, \tag{A.4}$$

where

$$\begin{aligned}
&\underline{1}_1 = x_1\bar{y}_1 + x_2\bar{y}_2 + x_3\bar{y}_3, && \underline{1}_2 = x_1\bar{y}_1 + \omega x_2\bar{y}_2 + \omega^2 x_3\bar{y}_3,\\
&\underline{1}_3 = x_1\bar{y}_1 + \omega^2 x_2\bar{y}_2 + \omega x_3\bar{y}_3, && \underline{1}_4 = x_1\bar{y}_2 + x_2\bar{y}_3 + x_3\bar{y}_1,\\
&\underline{1}_5 = x_1\bar{y}_2 + \omega x_2\bar{y}_3 + \omega^2 x_3\bar{y}_1, && \underline{1}_6 = x_1\bar{y}_2 + \omega^2 x_2\bar{y}_3 + \omega x_3\bar{y}_1,\\
&\underline{1}_7 = x_2\bar{y}_1 + x_3\bar{y}_2 + x_1\bar{y}_3, && \underline{1}_8 = x_2\bar{y}_1 + \omega^2 x_3\bar{y}_2 + \omega x_1\bar{y}_3,\\
&\underline{1}_9 = x_2\bar{y}_1 + \omega x_3\bar{y}_2 + \omega^2 x_1\bar{y}_3,
\end{aligned} \tag{A.5}$$

with $\omega = e^{2\pi i/3} \equiv -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. The singlets multiplications are given in Table 3.

It is worth mentioning that $\underline{3}\otimes\underline{3}\otimes\underline{3}$ has three invariants under the $\Delta(27)$ discrete group. Those invariants are $111+222+333$, $123+231+312-213-321-132$ and $123+231+312+213+321+132$. This is a good feature of the $\Delta(27)$ discrete group, that allows us to make invariant Yukawa couplings to generate fermion mass matrices.

Appendix B. The matrices of the 3 representation of $\Delta(27)$

The matrices of the $\Delta(27)$ triplet representation are given by:

$$C_1: \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},\ C_2: \begin{pmatrix} \omega & 0 & 0\\ 0 & \omega & 0\\ 0 & 0 & \omega\end{pmatrix},\ C_3: \begin{pmatrix} \omega^2 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega^2\end{pmatrix}, \tag{B.1}$$

$$C_4: \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ \omega & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \\ \omega^2 & 0 & 0 \end{pmatrix}, \tag{B.2}$$

$$C_5: \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega \\ \omega & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega^2 \\ \omega^2 & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix}, \tag{B.3}$$

$$C_6: \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{B.4}$$

$$C_7: \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega^2 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega^2 \\ 1 & 0 & 0 \end{pmatrix}, \tag{B.5}$$

$$C_8: \begin{pmatrix} 0 & 0 & 1 \\ \omega & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega \\ \omega^2 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \tag{B.6}$$

$$C_9: \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad \begin{pmatrix} \omega^2 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{B.7}$$

$$C_{10}: \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega^2 \\ \omega & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & 1 \\ \omega^2 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \omega^2 & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix}, \tag{B.8}$$

$$C_{11}: \begin{pmatrix} 0 & 0 & 1 \\ \omega^2 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega \\ 1 & 0 & 0 \\ 0 & \omega^2 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & \omega^2 \\ \omega & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{B.9}$$

Appendix C. The breaking patterns of $\Delta(27)$ by triplets

For $\Delta(27)$ triplets $\underline{3}$ we have the following VEV alignments:

(1) **The first alignment**: $(\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_3\rangle)$ then $\Delta(27)$ is broken into $\{e\} \equiv \{\text{identity}\}$, i.e., it is completely broken.
(2) **The second alignment**: $(\langle\phi_1\rangle, \langle\phi_1\rangle, \langle\phi_1\rangle)$ then $\Delta(27)$ is broken into Z_3 group which consisting of the elements $\{1, b, b^2\}$.
(3) **The third alignment**: $\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_2\rangle)$ or $(\langle\phi_1\rangle, \langle\phi_2\rangle, \langle\phi_1\rangle)$ or $(\langle\phi_1\rangle, \langle\phi_1\rangle, \langle\phi_3\rangle)$ then $\Delta(27)$ is completely broken.
(4) **The fourth alignment:** $(\langle\phi_1\rangle, 0, \langle\phi_3\rangle)$ or $(0, \langle\phi_2\rangle, \langle\phi_3\rangle)$ or $(\langle\phi_1\rangle, \langle\phi_2\rangle, 0)$ then $\Delta(27)$ is completely broken.
(5) **The fifth alignment:** $(\langle\phi_1\rangle, 0, \langle\phi_1\rangle)$ or $(0, \langle\phi_2\rangle, \langle\phi_2\rangle)$ or $(\langle\phi_1\rangle, \langle\phi_1\rangle, 0)$ then $\Delta(27)$ is completely broken.
(6) **The sixth alignment:** $(\langle\phi_1\rangle, 0, 0)$ or $(0, \langle\phi_2\rangle, 0)$ or $(0, 0, \langle\phi_3\rangle)$ then $\Delta(27)$ is broken into Z_3 groups, consisting of the elements $\{e, aa', (aa')^2\}$ or $\{e, a, a^2\}$ or $\{e, a', a'^2\}$, respectively.

Let us note that the breakings of $\Delta(27)$ under $\underline{3}$ and $\underline{\bar{3}}$ are the same.

References

[1] G. Aad, et al., ATLAS Collaboration, Phys. Lett. B 716 (2012) 1, http://dx.doi.org/10.1016/j.physletb.2012.08.020, arXiv:1207.7214 [hep-ex].

[2] S. Chatrchyan, et al., CMS Collaboration, Phys. Lett. B 716 (2012) 30, http://dx.doi.org/10.1016/j.physletb.2012.08.021, arXiv:1207.7235 [hep-ex].

[3] F.P. An, et al., Daya Bay Collaboration, Phys. Rev. Lett. 108 (2012) 171803, http://dx.doi.org/10.1103/PhysRevLett.108.171803, arXiv:1203.1669 [hep-ex].

[4] K. Abe, et al., T2K Collaboration, Phys. Rev. Lett. 107 (2011) 041801, http://dx.doi.org/10.1103/PhysRevLett.107.041801, arXiv:1106.2822 [hep-ex].

[5] P. Adamson, et al., MINOS Collaboration, Phys. Rev. Lett. 107 (2011) 181802, http://dx.doi.org/10.1103/PhysRevLett.107.181802, arXiv:1108.0015 [hep-ex].

[6] Y. Abe, et al., Double Chooz Collaboration, Phys. Rev. Lett. 108 (2012) 131801, http://dx.doi.org/10.1103/PhysRevLett.108.131801, arXiv:1112.6353 [hep-ex].

[7] J.K. Ahn, et al., RENO Collaboration, Phys. Rev. Lett. 108 (2012) 191802, http://dx.doi.org/10.1103/PhysRevLett.108.191802, arXiv:1204.0626 [hep-ex].

[8] D.V. Forero, M. Tortola, J.W.F. Valle, Phys. Rev. D 90 (9) (2014) 093006, http://dx.doi.org/10.1103/PhysRevD.90.093006, arXiv:1405.7540 [hep-ph].

[9] K.A. Olive, et al., Particle Data Group Collaboration, Chin. Phys. C 38 (2014) 090001, http://dx.doi.org/10.1088/1674-1137/38/9/090001.

[10] H. Fritzsch, Phys. Lett. B 70 (1977) 436, http://dx.doi.org/10.1016/0370-2693(77)90408-7.

[11] T. Fukuyama, H. Nishiura, arXiv:hep-ph/9702253.

[12] D.s. Du, Z.z. Xing, Phys. Rev. D 48 (1993) 2349, http://dx.doi.org/10.1103/PhysRevD.48.2349.

[13] R. Barbieri, G.R. Dvali, A. Strumia, Z. Berezhiani, L.J. Hall, Nucl. Phys. B 432 (1994) 49, http://dx.doi.org/10.1016/0550-3213(94)90593-2, arXiv:hep-ph/9405428.

[14] R.D. Peccei, K. Wang, Phys. Rev. D 53 (1996) 2712, http://dx.doi.org/10.1103/PhysRevD.53.2712, arXiv:hep-ph/9509242.

[15] H. Fritzsch, Z.Z. Xing, Prog. Part. Nucl. Phys. 45 (2000) 1, http://dx.doi.org/10.1016/S0146-6410(00)00102-2, arXiv:hep-ph/9912358.

[16] R.G. Roberts, A. Romanino, G.G. Ross, L. Velasco-Sevilla, Nucl. Phys. B 615 (2001) 358, http://dx.doi.org/10.1016/S0550-3213(01)00408-4, arXiv:hep-ph/0104088.

[17] H. Nishiura, K. Matsuda, T. Kikuchi, T. Fukuyama, Phys. Rev. D 65 (2002) 097301, http://dx.doi.org/10.1103/PhysRevD.65.097301, arXiv:hep-ph/0202189.

[18] I. de Medeiros Varzielas, G.G. Ross, Nucl. Phys. B 733 (2006) 31, http://dx.doi.org/10.1016/j.nuclphysb.2005.10.039, arXiv:hep-ph/0507176.

[19] A.E. Carcamo Hernandez, R. Martinez, J.A. Rodriguez, Eur. Phys. J. C 50 (2007) 935, http://dx.doi.org/10.1140/epjc/s10052-007-0264-0, arXiv:hep-ph/0606190.

[20] Y. Kajiyama, M. Raidal, A. Strumia, Phys. Rev. D 76 (2007) 117301, http://dx.doi.org/10.1103/PhysRevD.76.117301, arXiv:0705.4559 [hep-ph].

[21] A.E. Carcamo Hernandez, R. Rahman, arXiv:1007.0447 [hep-ph].

[22] G.C. Branco, D. Emmanuel-Costa, C. Simoes, Phys. Lett. B 690 (2010) 62, http://dx.doi.org/10.1016/j.physletb.2010.05.009, arXiv:1001.5065 [hep-ph].

[23] P. Leser, H. Pas, Phys. Rev. D 84 (2011) 017303, http://dx.doi.org/10.1103/PhysRevD.84.017303, arXiv:1104.2448 [hep-ph].

[24] M. Gupta, G. Ahuja, Int. J. Mod. Phys. A 27 (2012) 1230033, http://dx.doi.org/10.1142/S0217751X12300335, arXiv:1302.4823 [hep-ph].

[25] A.E. Carcamo Hernandez, R. Martinez, F. Ochoa, Phys. Rev. D 87 (7) (2013) 075009, http://dx.doi.org/10.1103/PhysRevD.87.075009, arXiv:1302.1757 [hep-ph].

[26] H. Päs, E. Schumacher, Phys. Rev. D 89 (9) (2014) 096010, http://dx.doi.org/10.1103/PhysRevD.89.096010, arXiv:1401.2328 [hep-ph].

[27] A.E. Carcamo Hernandez, S. Kovalenko, I. Schmidt, arXiv:1411.2913 [hep-ph].

[28] A.E.C. Hernández, I.d.M. Varzielas, J. Phys. G 42 (6) (2015) 065002, http://dx.doi.org/10.1088/0954-3899/42/6/065002, arXiv:1410.2481 [hep-ph].

[29] H. Nishiura, T. Fukuyama, Mod. Phys. Lett. A 29 (2014) 0147, http://dx.doi.org/10.1142/S0217732314501478, arXiv:1405.2416 [hep-ph].

[30] M. Frank, C. Hamzaoui, N. Pourtolami, M. Toharia, Phys. Lett. B 742 (2015) 178, http://dx.doi.org/10.1016/j.physletb.2015.01.025, arXiv:1406.2331 [hep-ph].
[31] R. Sinha, R. Samanta, A. Ghosal, arXiv:1508.05227 [hep-ph].
[32] H. Nishiura, T. Fukuyama, Phys. Lett. B 753 (2016) 57, http://dx.doi.org/10.1016/j.physletb.2015.11.080, arXiv:1510.01035 [hep-ph].
[33] R.R. Gautam, M. Singh, M. Gupta, Phys. Rev. D 92 (1) (2015) 013006, http://dx.doi.org/10.1103/PhysRevD.92.013006, arXiv:1506.04868 [hep-ph].
[34] H. Päs, E. Schumacher, Phys. Rev. D 92 (11) (2015) 114025, http://dx.doi.org/10.1103/PhysRevD.92.114025, arXiv:1510.08757 [hep-ph].
[35] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, M. Tanimoto, Prog. Theor. Phys. Suppl. 183 (2010) 1, http://dx.doi.org/10.1143/PTPS.183.1, arXiv:1003.3552 [hep-th].
[36] G. Altarelli, F. Feruglio, Rev. Mod. Phys. 82 (2010) 2701, http://dx.doi.org/10.1103/RevModPhys.82.2701, arXiv:1002.0211 [hep-ph].
[37] S.F. King, C. Luhn, Rep. Prog. Phys. 76 (2013) 056201, http://dx.doi.org/10.1088/0034-4885/76/5/056201, arXiv:1301.1340 [hep-ph].
[38] S.F. King, A. Merle, S. Morisi, Y. Shimizu, M. Tanimoto, New J. Phys. 16 (2014) 045018, http://dx.doi.org/10.1088/1367-2630/16/4/045018, arXiv:1402.4271 [hep-ph].
[39] E. Ma, G. Rajasekaran, Phys. Rev. D 64 (2001) 113012, http://dx.doi.org/10.1103/PhysRevD.64.113012, arXiv:hep-ph/0106291.
[40] X.G. He, Y.Y. Keum, R.R. Volkas, JHEP 0604 (2006) 039, http://dx.doi.org/10.1088/1126-6708/2006/04/039, arXiv:hep-ph/0601001.
[41] M.C. Chen, S.F. King, JHEP 0906 (2009) 072, http://dx.doi.org/10.1088/1126-6708/2009/06/072, arXiv:0903.0125 [hep-ph].
[42] P.V. Dong, L.T. Hue, H.N. Long, D.V. Soa, Phys. Rev. D 81 (2010) 053004, http://dx.doi.org/10.1103/PhysRevD.81.053004, arXiv:1001.4625 [hep-ph].
[43] Y.H. Ahn, S.K. Kang, Phys. Rev. D 86 (2012) 093003, http://dx.doi.org/10.1103/PhysRevD.86.093003, arXiv:1203.4185 [hep-ph].
[44] N. Memenga, W. Rodejohann, H. Zhang, Phys. Rev. D 87 (5) (2013) 053021, http://dx.doi.org/10.1103/PhysRevD.87.053021, arXiv:1301.2963 [hep-ph].
[45] R. Gonzalez Felipe, H. Serodio, J.P. Silva, Phys. Rev. D 88 (1) (2013) 015015, http://dx.doi.org/10.1103/PhysRevD.88.015015, arXiv:1304.3468 [hep-ph].
[46] I. de Medeiros Varzielas, D. Pidt, JHEP 1303 (2013) 065, http://dx.doi.org/10.1007/JHEP03(2013)065, arXiv:1211.5370 [hep-ph].
[47] H. Ishimori, E. Ma, Phys. Rev. D 86 (2012) 045030, http://dx.doi.org/10.1103/PhysRevD.86.045030, arXiv:1205.0075 [hep-ph].
[48] S.F. King, S. Morisi, E. Peinado, J.W.F. Valle, Phys. Lett. B 724 (2013) 68, http://dx.doi.org/10.1016/j.physletb.2013.05.067, arXiv:1301.7065 [hep-ph].
[49] A.E. Carcamo Hernandez, I. de Medeiros Varzielas, S.G. Kovalenko, H. Päs, I. Schmidt, Phys. Rev. D 88 (7) (2013) 076014, http://dx.doi.org/10.1103/PhysRevD.88.076014, arXiv:1307.6499 [hep-ph].
[50] K.S. Babu, E. Ma, J.W.F. Valle, Phys. Lett. B 552 (2003) 207, http://dx.doi.org/10.1016/S0370-2693(02)03153-2, arXiv:hep-ph/0206292.
[51] G. Altarelli, F. Feruglio, Nucl. Phys. B 741 (2006) 215, http://dx.doi.org/10.1016/j.nuclphysb.2006.02.015, arXiv:hep-ph/0512103.
[52] S. Morisi, M. Nebot, K.M. Patel, E. Peinado, J.W.F. Valle, Phys. Rev. D 88 (2013) 036001, http://dx.doi.org/10.1103/PhysRevD.88.036001, arXiv:1303.4394 [hep-ph].
[53] G. Altarelli, F. Feruglio, Nucl. Phys. B 720 (2005) 64, http://dx.doi.org/10.1016/j.nuclphysb.2005.05.005, arXiv:hep-ph/0504165.
[54] A. Kadosh, E. Pallante, JHEP 1008 (2010) 115, http://dx.doi.org/10.1007/JHEP08(2010)115, arXiv:1004.0321 [hep-ph].
[55] A. Kadosh, JHEP 1306 (2013) 114, http://dx.doi.org/10.1007/JHEP06(2013)114, arXiv:1303.2645 [hep-ph].
[56] F. del Aguila, A. Carmona, J. Santiago, JHEP 1008 (2010) 127, http://dx.doi.org/10.1007/JHEP08(2010)127, arXiv:1001.5151 [hep-ph].
[57] M.D. Campos, A.E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, E. Schumacher, Phys. Rev. D 90 (1) (2014) 016006, http://dx.doi.org/10.1103/PhysRevD.90.016006, arXiv:1403.2525 [hep-ph].
[58] V.V. Vien, H.N. Long, Int. J. Mod. Phys. A 30 (21) (2015) 1550117, http://dx.doi.org/10.1142/S0217751X15501171, arXiv:1405.4665 [hep-ph].

[59] A.E. Cárcamo Hernández, R. Martinez, Nucl. Phys. B 905 (2016) 337, http://dx.doi.org/10.1016/j.nuclphysb.2016.02.025, arXiv:1501.05937 [hep-ph].
[60] C.C. Nishi, Phys. Rev. D 93 (2016) 093009, arXiv:1601.00977 [hep-ph].
[61] A.E.C. Hernández, Eur. Phys. J. C 76 (9) (2016) 503, arXiv:1512.09092 [hep-ph].
[62] S.L. Chen, M. Frigerio, E. Ma, Phys. Rev. D 70 (2004) 073008, http://dx.doi.org/10.1103/PhysRevD.70.073008, arXiv:hep-ph/0404084, Erratum: Phys. Rev. D 70 (2004) 079905, http://dx.doi.org/10.1103/PhysRevD.70.079905.
[63] P.V. Dong, H.N. Long, C.H. Nam, V.V. Vien, Phys. Rev. D 85 (2012) 053001, http://dx.doi.org/10.1103/PhysRevD.85.053001, arXiv:1111.6360 [hep-ph].
[64] G. Bhattacharyya, P. Leser, H. Pas, Phys. Rev. D 83 (2011) 011701, http://dx.doi.org/10.1103/PhysRevD.83.011701, arXiv:1006.5597 [hep-ph].
[65] A.G. Dias, A.C.B. Machado, C.C. Nishi, Phys. Rev. D 86 (2012) 093005, http://dx.doi.org/10.1103/PhysRevD.86.093005, arXiv:1206.6362 [hep-ph].
[66] D. Meloni, JHEP 1205 (2012) 124, http://dx.doi.org/10.1007/JHEP05(2012)124, arXiv:1203.3126 [hep-ph].
[67] F. González Canales, A. Mondragón, M. Mondragón, U.J. Saldaña Salazar, L. Velasco-Sevilla, Phys. Rev. D 88 (2013) 096004, http://dx.doi.org/10.1103/PhysRevD.88.096004, arXiv:1304.6644 [hep-ph].
[68] E. Ma, B. Melic, Phys. Lett. B 725 (2013) 402, http://dx.doi.org/10.1016/j.physletb.2013.07.015, arXiv:1303.6928 [hep-ph].
[69] Y. Kajiyama, H. Okada, K. Yagyu, Nucl. Phys. B 887 (2014) 358, http://dx.doi.org/10.1016/j.nuclphysb.2014.08.009, arXiv:1309.6234 [hep-ph].
[70] A.E. Cárcamo Hernández, R. Martínez, F. Ochoa, arXiv:1309.6567 [hep-ph].
[71] E. Ma, R. Srivastava, Phys. Lett. B 741 (2015) 217, http://dx.doi.org/10.1016/j.physletb.2014.12.049, arXiv:1411.5042 [hep-ph].
[72] A.E.C. Hernández, R. Martinez, J. Nisperuza, Eur. Phys. J. C 75 (2) (2015) 72, http://dx.doi.org/10.1140/epjc/s10052-015-3278-z, arXiv:1401.0937 [hep-ph].
[73] A.E.C. Hernández, E.C. Mur, R. Martinez, Phys. Rev. D 90 (7) (2014) 073001, http://dx.doi.org/10.1103/PhysRevD.90.073001, arXiv:1407.5217 [hep-ph].
[74] A.E.C. Hernández, I.d.M. Varzielas, E. Schumacher, Phys. Rev. D 93 (2016) 016003, http://dx.doi.org/10.1103/PhysRevD.93.016003, arXiv:1509.02083 [hep-ph].
[75] A.E.C. Hernández, I.d.M. Varzielas, N.A. Neill, arXiv:1511.07420 [hep-ph].
[76] A.E.C. Hernández , I.d.M. Varzielas, E. Schumacher, arXiv:1601.00661 [hep-ph].
[77] R.N. Mohapatra, C.C. Nishi, Phys. Rev. D 86 (2012) 073007, http://dx.doi.org/10.1103/PhysRevD.86.073007, arXiv:1208.2875 [hep-ph].
[78] I. de Medeiros Varzielas, L. Lavoura, J. Phys. G 40 (2013) 085002, http://dx.doi.org/10.1088/0954-3899/40/8/085002, arXiv:1212.3247 [hep-ph].
[79] G.J. Ding, S.F. King, C. Luhn, A.J. Stuart, JHEP 1305 (2013) 084, http://dx.doi.org/10.1007/JHEP05(2013)084, arXiv:1303.6180 [hep-ph].
[80] H. Ishimori, Y. Shimizu, M. Tanimoto, A. Watanabe, Phys. Rev. D 83 (2011) 033004, http://dx.doi.org/10.1103/PhysRevD.83.033004, arXiv:1010.3805 [hep-ph].
[81] G.J. Ding, Y.L. Zhou, Nucl. Phys. B 876 (2013) 418, http://dx.doi.org/10.1016/j.nuclphysb.2013.08.011, arXiv:1304.2645 [hep-ph].
[82] C. Hagedorn, M. Serone, JHEP 1110 (2011) 083, http://dx.doi.org/10.1007/JHEP10(2011)083, arXiv:1106.4021 [hep-ph].
[83] M.D. Campos, A.E.C. Hernández, H. Päs, E. Schumacher, Phys. Rev. D 91 (11) (2015) 116011, http://dx.doi.org/10.1103/PhysRevD.91.116011, arXiv:1408.1652 [hep-ph].
[84] P.V. Dong, H.N. Long, D.V. Soa, V.V. Vien, Eur. Phys. J. C 71 (2011) 1544, http://dx.doi.org/10.1140/epjc/s10052-011-1544-2, arXiv:1009.2328 [hep-ph].
[85] V.V. Vien, H.N. Long, D.P. Khoi, Int. J. Mod. Phys. A 30 (17) (2015) 1550102, http://dx.doi.org/10.1142/S0217751X1550102X, arXiv:1506.06063 [hep-ph].
[86] P.H. Frampton, T.W. Kephart, Int. J. Mod. Phys. A 10 (1995) 4689, http://dx.doi.org/10.1142/S0217751X95002187, arXiv:hep-ph/9409330.
[87] W. Grimus, L. Lavoura, Phys. Lett. B 572 (2003) 189, http://dx.doi.org/10.1016/j.physletb.2003.08.032, arXiv:hep-ph/0305046.
[88] W. Grimus, A.S. Joshipura, S. Kaneko, L. Lavoura, M. Tanimoto, JHEP 0407 (2004) 078, http://dx.doi.org/10.1088/1126-6708/2004/07/078, arXiv:hep-ph/0407112.
[89] M. Frigerio, S. Kaneko, E. Ma, M. Tanimoto, Phys. Rev. D 71 (2005) 011901, http://dx.doi.org/10.1103/PhysRevD.71.011901, arXiv:hep-ph/0409187.

[90] K.S. Babu, J. Kubo, Phys. Rev. D 71 (2005) 056006, http://dx.doi.org/10.1103/PhysRevD.71.056006, arXiv:hep-ph/0411226.

[91] A. Adulpravitchai, A. Blum, C. Hagedorn, JHEP 0903 (2009) 046, http://dx.doi.org/10.1088/1126-6708/2009/03/046, arXiv:0812.3799 [hep-ph].

[92] H. Ishimori, T. Kobayashi, H. Ohki, Y. Omura, R. Takahashi, M. Tanimoto, Phys. Lett. B 662 (2008) 178, http://dx.doi.org/10.1016/j.physletb.2008.03.007, arXiv:0802.2310 [hep-ph].

[93] C. Hagedorn, R. Ziegler, Phys. Rev. D 82 (2010) 053011, http://dx.doi.org/10.1103/PhysRevD.82.053011, arXiv:1007.1888 [hep-ph].

[94] D. Meloni, S. Morisi, E. Peinado, Phys. Lett. B 703 (2011) 281, http://dx.doi.org/10.1016/j.physletb.2011.07.084, arXiv:1104.0178 [hep-ph].

[95] V.V. Vien, H.N. Long, Int. J. Mod. Phys. A 28 (2013) 1350159, http://dx.doi.org/10.1142/S0217751X13501595, arXiv:1312.5034 [hep-ph].

[96] C. Luhn, S. Nasri, P. Ramond, Phys. Lett. B 652 (2007) 27, http://dx.doi.org/10.1016/j.physletb.2007.06.059, arXiv:0706.2341 [hep-ph].

[97] C. Hagedorn, M.A. Schmidt, A.Y. Smirnov, Phys. Rev. D 79 (2009) 036002, http://dx.doi.org/10.1103/PhysRevD.79.036002, arXiv:0811.2955 [hep-ph].

[98] Q.H. Cao, S. Khalil, E. Ma, H. Okada, Phys. Rev. Lett. 106 (2011) 131801, http://dx.doi.org/10.1103/PhysRevLett.106.131801, arXiv:1009.5415 [hep-ph].

[99] C. Luhn, K.M. Parattu, A. Wingerter, JHEP 1212 (2012) 096, http://dx.doi.org/10.1007/JHEP12(2012)096, arXiv:1210.1197 [hep-ph].

[100] Y. Kajiyama, H. Okada, K. Yagyu, JHEP 1310 (2013) 196, http://dx.doi.org/10.1007/JHEP10(2013)196, arXiv:1307.0480 [hep-ph].

[101] C. Bonilla, S. Morisi, E. Peinado, J.W.F. Valle, Phys. Lett. B 742 (2015) 99, http://dx.doi.org/10.1016/j.physletb.2015.01.017, arXiv:1411.4883 [hep-ph].

[102] V.V. Vien, H.N. Long, JHEP 1404 (2014) 133, http://dx.doi.org/10.1007/JHEP04(2014)133, arXiv:1402.1256 [hep-ph].

[103] V.V. Vien, Mod. Phys. Lett. A 29 (2014) 28, http://dx.doi.org/10.1142/S0217732314501399, arXiv:1508.02585 [hep-ph].

[104] A.E.C. Hernández, R. Martinez, arXiv:1501.07261 [hep-ph].

[105] C. Arbeláez, A.E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, Phys. Rev. D 92 (11) (2015) 115015, http://dx.doi.org/10.1103/PhysRevD.92.115015, arXiv:1507.03852 [hep-ph].

[106] G.J. Ding, Nucl. Phys. B 853 (2011) 635, http://dx.doi.org/10.1016/j.nuclphysb.2011.08.012, arXiv:1105.5879 [hep-ph].

[107] C. Hartmann, Phys. Rev. D 85 (2012) 013012, http://dx.doi.org/10.1103/PhysRevD.85.013012, arXiv:1109.5143 [hep-ph].

[108] C. Hartmann, A. Zee, Nucl. Phys. B 853 (2011) 105, http://dx.doi.org/10.1016/j.nuclphysb.2011.07.023, arXiv:1106.0333 [hep-ph].

[109] Y. Kajiyama, H. Okada, Nucl. Phys. B 848 (2011) 303, http://dx.doi.org/10.1016/j.nuclphysb.2011.02.020, arXiv:1011.5753 [hep-ph].

[110] A. Aranda, C.D. Carone, R.F. Lebed, Phys. Rev. D 62 (2000) 016009, http://dx.doi.org/10.1103/PhysRevD.62.016009, arXiv:hep-ph/0002044.

[111] A. Aranda, Phys. Rev. D 76 (2007) 111301, http://dx.doi.org/10.1103/PhysRevD.76.111301, arXiv:0707.3661 [hep-ph].

[112] M.C. Chen, K.T. Mahanthappa, Phys. Lett. B 652 (2007) 34, http://dx.doi.org/10.1016/j.physletb.2007.06.064, arXiv:0705.0714 [hep-ph].

[113] P.H. Frampton, T.W. Kephart, S. Matsuzaki, Phys. Rev. D 78 (2008) 073004, http://dx.doi.org/10.1103/PhysRevD.78.073004, arXiv:0807.4713 [hep-ph].

[114] D.A. Eby, P.H. Frampton, X.G. He, T.W. Kephart, Phys. Rev. D 84 (2011) 037302, http://dx.doi.org/10.1103/PhysRevD.84.037302, arXiv:1103.5737 [hep-ph].

[115] P.H. Frampton, C.M. Ho, T.W. Kephart, Phys. Rev. D 89 (2) (2014) 027701, http://dx.doi.org/10.1103/PhysRevD.89.027701, arXiv:1305.4402 [hep-ph].

[116] I. de Medeiros Varzielas, D. Emmanuel-Costa, P. Leser, Phys. Lett. B 716 (2012) 193, http://dx.doi.org/10.1016/j.physletb.2012.08.008, arXiv:1204.3633 [hep-ph].

[117] G. Bhattacharyya, I. de Medeiros Varzielas, P. Leser, Phys. Rev. Lett. 109 (2012) 241603, http://dx.doi.org/10.1103/PhysRevLett.109.241603, arXiv:1210.0545 [hep-ph].

[118] E. Ma, Phys. Lett. B 723 (2013) 161, http://dx.doi.org/10.1016/j.physletb.2013.05.011, arXiv:1304.1603 [hep-ph].

[119] C.C. Nishi, Phys. Rev. D 88 (3) (2013) 033010, http://dx.doi.org/10.1103/PhysRevD.88.033010, arXiv:1306.0877 [hep-ph].
[120] I. de Medeiros Varzielas, D. Pidt, J. Phys. G 41 (2014) 025004, http://dx.doi.org/10.1088/0954-3899/41/2/025004, arXiv:1307.0711 [hep-ph].
[121] A. Aranda, C. Bonilla, S. Morisi, E. Peinado, J.W.F. Valle, Phys. Rev. D 89 (3) (2014) 033001, http://dx.doi.org/10.1103/PhysRevD.89.033001, arXiv:1307.3553 [hep-ph].
[122] I. de Medeiros Varzielas, JHEP 1508 (2015) 157, http://dx.doi.org/10.1007/JHEP08(2015)157, arXiv:1507.00338 [hep-ph].
[123] P. Chen, G.J. Ding, A.D. Rojas, C.A. Vaquera-Araujo, J.W.F. Valle, JHEP 1601 (2016) 007, http://dx.doi.org/10.1007/JHEP01(2016)007, arXiv:1509.06683 [hep-ph].
[124] H. Georgi, A. Pais, Phys. Rev. D 19 (1979) 2746, http://dx.doi.org/10.1103/PhysRevD.19.2746.
[125] J.W.F. Valle, M. Singer, Phys. Rev. D 28 (1983) 540, http://dx.doi.org/10.1103/PhysRevD.28.540.
[126] F. Pisano, V. Pleitez, Phys. Rev. D 46 (1992) 410, http://dx.doi.org/10.1103/PhysRevD.46.410, arXiv:hep-ph/9206242.
[127] R. Foot, O.F. Hernandez, F. Pisano, V. Pleitez, Phys. Rev. D 47 (1993) 4158, http://dx.doi.org/10.1103/PhysRevD.47.4158, arXiv:hep-ph/9207264.
[128] P.H. Frampton, Phys. Rev. Lett. 69 (1992) 2889, http://dx.doi.org/10.1103/PhysRevLett.69.2889.
[129] D. Ng, Phys. Rev. D 49 (1994) 4805, http://dx.doi.org/10.1103/PhysRevD.49.4805, arXiv:hep-ph/9212284.
[130] T.V. Duong, E. Ma, Phys. Lett. B 316 (1993) 307, http://dx.doi.org/10.1016/0370-2693(93)90329-G, arXiv:hep-ph/9306264.
[131] H.N. Long, Phys. Rev. D 54 (1996) 4691, http://dx.doi.org/10.1103/PhysRevD.54.4691, arXiv:hep-ph/9607439.
[132] H.N. Long, Phys. Rev. D 53 (1996) 437, http://dx.doi.org/10.1103/PhysRevD.53.437, arXiv:hep-ph/9504274.
[133] R. Foot, H.N. Long, T.A. Tran, Phys. Rev. D 50 (1994) R34, http://dx.doi.org/10.1103/PhysRevD.50.R34, arXiv:hep-ph/9402243.
[134] R.A. Diaz, R. Martinez, F. Ochoa, Phys. Rev. D 69 (2004) 095009, http://dx.doi.org/10.1103/PhysRevD.69.095009, arXiv:hep-ph/0309280.
[135] R.A. Diaz, R. Martinez, F. Ochoa, Phys. Rev. D 72 (2005) 035018, http://dx.doi.org/10.1103/PhysRevD.72.035018, arXiv:hep-ph/0411263.
[136] A.G. Dias, R. Martinez, V. Pleitez, Eur. Phys. J. C 39 (2005) 101, http://dx.doi.org/10.1140/epjc/s2004-02083-0, arXiv:hep-ph/0407141.
[137] F. Ochoa, R. Martinez, Phys. Rev. D 72 (2005) 035010, http://dx.doi.org/10.1103/PhysRevD.72.035010, arXiv:hep-ph/0505027.
[138] A.E. Carcamo Hernandez, R. Martinez, F. Ochoa, Phys. Rev. D 73 (2006) 035007, http://dx.doi.org/10.1103/PhysRevD.73.035007, arXiv:hep-ph/0510421.
[139] C. Alvarado, R. Martinez, F. Ochoa, Phys. Rev. D 86 (2012) 025027, http://dx.doi.org/10.1103/PhysRevD.86.025027, arXiv:1207.0014 [hep-ph].
[140] M.E. Catano, R. Martinez, F. Ochoa, Phys. Rev. D 86 (2012) 073015, http://dx.doi.org/10.1103/PhysRevD.86.073015, arXiv:1206.1966 [hep-ph].
[141] S.M. Boucenna, S. Morisi, J.W.F. Valle, Phys. Rev. D 90 (1) (2014) 013005, http://dx.doi.org/10.1103/PhysRevD.90.013005, arXiv:1405.2332 [hep-ph].
[142] S.M. Boucenna, R.M. Fonseca, F. Gonzalez-Canales, J.W.F. Valle, Phys. Rev. D 91 (3) (2015) 031702, http://dx.doi.org/10.1103/PhysRevD.91.031702, arXiv:1411.0566 [hep-ph].
[143] V.Q. Phong, H.N. Long, V.T. Van, L.H. Minh, Eur. Phys. J. C 75 (7) (2015) 342, http://dx.doi.org/10.1140/epjc/s10052-015-3550-2, arXiv:1409.0750 [hep-ph].
[144] S.M. Boucenna, J.W.F. Valle, A. Vicente, Phys. Rev. D 92 (5) (2015) 053001, http://dx.doi.org/10.1103/PhysRevD.92.053001, arXiv:1502.07546 [hep-ph].
[145] G. De Conto, A.C.B. Machado, V. Pleitez, Phys. Rev. D 92 (7) (2015) 075031, http://dx.doi.org/10.1103/PhysRevD.92.075031, arXiv:1505.01343 [hep-ph].
[146] F.C. Correia, V. Pleitez, Phys. Rev. D 92 (2015) 113006, http://dx.doi.org/10.1103/PhysRevD.92.113006, arXiv:1508.07319 [hep-ph].
[147] H. Okada, N. Okada, Y. Orikasa, Phys. Rev. D 93 (2016) 073006, http://dx.doi.org/10.1103/PhysRevD.93.073006, arXiv:1504.01204 [hep-ph].
[148] H.N. Long, arXiv:1504.06908 [hep-ph].
[149] H.N. Long, Phys. Int. 7 (1) (2016) 1, arXiv:1501.01852 [hep-ph].
[150] D.T. Binh, D.T. Huong, H.N. Long, Zh. Eksp. Teor. Fiz. 148 (2015) 1115, http://dx.doi.org/10.7868/S004445101512007X, arXiv:1504.03510 [hep-ph].

[151] L.T. Hue, H.N. Long, T.T. Thuc, N.T. Phong, Nucl. Phys. B 907 (2016) 37, http://dx.doi.org/10.1016/j.nuclphysb.2016.03.034, arXiv:1512.03266 [hep-ph].
[152] P.B. Pal, Phys. Rev. D 52 (1995) 1659, http://dx.doi.org/10.1103/PhysRevD.52.1659, arXiv:hep-ph/9411406.
[153] J.K. Mizukoshi, C.A. de S. Pires, F.S. Queiroz, P.S. Rodrigues da Silva, Phys. Rev. D 83 (2011) 065024, http://dx.doi.org/10.1103/PhysRevD.83.065024, arXiv:1010.4097 [hep-ph].
[154] S.M. Boucenna, S. Morisi, A. Vicente, arXiv:1512.06878 [hep-ph].
[155] A.E.C. Hernández, I. Nisandzic, arXiv:1512.07165 [hep-ph].
[156] P.V. Dong, N.T.K. Ngan, arXiv:1512.09073 [hep-ph].
[157] F. Pisano, V. Pleitez, Phys. Rev. D 46 (1992) 410, arXiv:hep-ph/9206242.
[158] P.H. Frampton, Phys. Rev. Lett. 69 (1992) 2889.
[159] R. Foot, et al., Phys. Rev. D 47 (1993) 4158.
[160] D. Chang, H.N. Long, Phys. Rev. D 73 (2006) 053006, http://dx.doi.org/10.1103/PhysRevD.73.053006, arXiv:hep-ph/0603098.
[161] P.F. Harrison, D.H. Perkins, W.G. Scott, Phys. Lett. B 530 (2002) 167, http://dx.doi.org/10.1016/S0370-2693(02)01336-9, arXiv:hep-ph/0202074.
[162] M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, JHEP 1411 (2014) 052, http://dx.doi.org/10.1007/JHEP11(2014)052, arXiv:1409.5439 [hep-ph].
[163] J.M. Conrad, Phys. Scr. T 158 (2013) 014012, http://dx.doi.org/10.1088/0031-8949/2013/T158/014012, arXiv:1310.0108 [hep-ex].
[164] S. Parke, Phys. Scr. T 158 (2013) 014013, http://dx.doi.org/10.1088/0031-8949/2013/T158/014013, arXiv:1310.5992 [hep-ph].
[165] M. Tegmark, et al., SDSS Collaboration, Phys. Rev. D 69 (2004) 103501, http://dx.doi.org/10.1103/PhysRevD.69.103501, arXiv:astro-ph/0310723.
[166] T.J. Weiler, arXiv:1308.1715 [hep-ph].
[167] M. Auger, et al., EXO-200 Collaboration, Phys. Rev. Lett. 109 (2012) 032505, http://dx.doi.org/10.1103/PhysRevLett.109.032505, arXiv:1205.5608 [hep-ex].
[168] I. Abt, et al., arXiv:hep-ex/0404039.
[169] K.H. Ackermann, et al., GERDA Collaboration, Eur. Phys. J. C 73 (3) (2013) 2330, http://dx.doi.org/10.1140/epjc/s10052-013-2330-0, arXiv:1212.4067 [physics.ins-det].
[170] F. Alessandria, et al., arXiv:1109.0494 [nucl-ex].
[171] A. Gando, et al., KamLAND-Zen Collaboration, Phys. Rev. C 85 (2012) 045504, http://dx.doi.org/10.1103/PhysRevC.85.045504, arXiv:1201.4664 [hep-ex].
[172] J.B. Albert, et al., EXO-200 Collaboration, Phys. Rev. D 90 (9) (2014) 092004, http://dx.doi.org/10.1103/PhysRevD.90.092004, arXiv:1409.6829 [hep-ex].
[173] C.E. Aalseth, et al., Majorana Collaboration, Nucl. Phys. B, Proc. Suppl. 217 (2011) 44, http://dx.doi.org/10.1016/j.nuclphysbps.2011.04.063, arXiv:1101.0119 [nucl-ex].
[174] S.M. Bilenky, C. Giunti, Int. J. Mod. Phys. A 30 (04n05) (2015) 1530001, http://dx.doi.org/10.1142/S0217751X1530001X, arXiv:1411.4791 [hep-ph].
[175] H. Okada, N. Okada, Y. Orikasa, K. Yagyu, Phys. Rev. D 94 (1) (2016) 015002, http://dx.doi.org/10.1103/PhysRevD.94.015002, arXiv:1604.01948 [hep-ph].